Case Study

Shock Wave Dynamics in a CD Nozzle using Ansys Fluent: A Problem-Based Learning Case Study

Developed and curated by the Academic Development Team

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Ansys Software Used

This resource uses Ansys Fluent® fluid simulation software, Mechanical™ structural finite element analysis software, Ansys Discovery™ 3D product simulation software,and Ansys Ensight™ simulation data visualization software

Summary

This case study, developed as a Problem-Based Learning (PBL) exercise, investigates the complex flow characteristics within a 2D convergent-divergent (CD) nozzle, with a specific focus on the occurrence and impact of normal shock waves. The nozzle, featuring an inlet radius of 250mm, an exit radius of 290mm, and a total length of 850mm, operates with an inlet Mach number of 0.5 and an inlet pressure of 250000 Pa. The core problem, central to this PBL approach, is to determine the critical throat radius required for choked flow and to analyze the corresponding exit Mach numbers under both subsonic and supersonic exit conditions, explicitly considering the presence of a normal shock wave within the diverging section. This study leverages fundamental normal shock theory and isentropic flow relations to predict and analyze these critical flow phenomena.

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1. Problem Statement

The objective of this case study is to analyze the flow within a 2D CD nozzle with the following specifications:

Inlet Radius (R_{in}): 250 mm
Exit Radius (R_{exit}): 290 mm
Total Length (L): 850 mm
Inlet Mach Number (M_{in}): 0.5
Inlet Pressure (P_{in}): 250,000 Pa
The problem requires determination of:

Critical Throat Radius (R_{throat}): The radius of the nozzle throat required to achieve choked flow conditions (Mach number of 1 at the throat). This will be calculated using isentropic flow relations from the inlet conditions.

Exit Mach Number (Subsonic Exit): The Mach number at the nozzle exit when a normal shock wave is present within the diverging section, resulting in a subsonic flow at the exit.

Exit Mach Number (Supersonic Exit): The Mach number at the nozzle exit when no shock wave is present and the flow remains supersonic throughout the diverging section.

All calculations will leverage the normal shock equations and isentropic flow relations, assuming air as the working fluid (γ=1.4).

2. Normal Shock Theory

A normal shock wave represents a discontinuity within a supersonic flow field, characterized by an exceedingly thin region (approximating a few mean free paths of the constituent molecules) where the flow properties undergo nearly instantaneous changes. It is oriented perpendicularly to the direction of the incident flow and constitutes the strongest possible shock wave for a given upstream Mach number, consequently incurring the most substantial thermodynamic losses.

2.1 Assumptions for Analysis:

For the derivation of the governing equations pertaining to a normal shock, the following simplifying assumptions are typically employed. These assumptions allow for the analytical tractability of the problem while still capturing the essential physics of the phenomenon:

Steady Flow: All flow properties at any given spatial location remain invariant with respect to time. This simplifies the conservation equations by eliminating time-derivative terms.

One-Dimensional Flow: Flow properties are considered to vary exclusively in the direction normal to the shock front. This reduces the complexity of the problem from three dimensions to one, making analytical solutions feasible. While simplification, it provides a strong foundation for understanding the core physics.

Adiabatic Process: No heat transfer occurs into or out of the control volume encompassing the shock. This implies that the total energy of the fluid remains constant, although its distribution between kinetic and internal energy changes.

No External Work: No work is performed by or upon the fluid as it traverses the shock. This further simplifies the energy equation, as there are no shaft work or electrical work terms.

Negligible Viscosity and Thermal Conduction: Although these are the inherent physical mechanisms responsible for shock formation and the associated entropy increase, for macroscopic analysis, the shock is treated as an infinitesimally thin discontinuity wherein these dissipative effects are localized. This allows for the application of macroscopic conservation laws across the shock without needing to resolve the complex internal structure.

Ideal Gas Behavior: The working fluid (air) is assumed to behave as an ideal gas, adhering to the equation of state P=ρRT. This simplifies the thermodynamic relationships and allows for straightforward calculations of properties.

Constant Specific Heats: The specific heats at constant pressure (c_p) and constant volume (c_v), and their ratio $\gamma = c_p/c_v$, are considered constant throughout the process. This is a common simplification for air at moderate temperatures.

2.2 Governing Equations for a shock wave:

To analyze these changes across the normal shock, we rely on the fundamental conservation laws of fluid dynamics, applied across the shock in a control volume that is stationary relative to the shock front. We consider a steady, one-dimensional flow of an ideal gas with constant specific heats.

2.2.1 Conservation of Mass (Continuity Equation)

This principle states that mass is neither created nor destroyed. For a steady flow across a normal shock, the mass flow rate entering the shock must be equal to the mass flow rate exiting the shock.
Understanding: The product of the fluid's density (ρ) and its velocity (u) perpendicular to the shock front remains constant. This means if the velocity changes, the density must change inversely to maintain the same mass flow.
Equation:

$$\rho_1 u_1 = \rho_2 u_2$$

Where:

$\rho_2,\ u_2 \rho_1,\ u_1$: Density and velocity upstream of the shock (State 1)

: Density and velocity downstream of the shock (State 2)

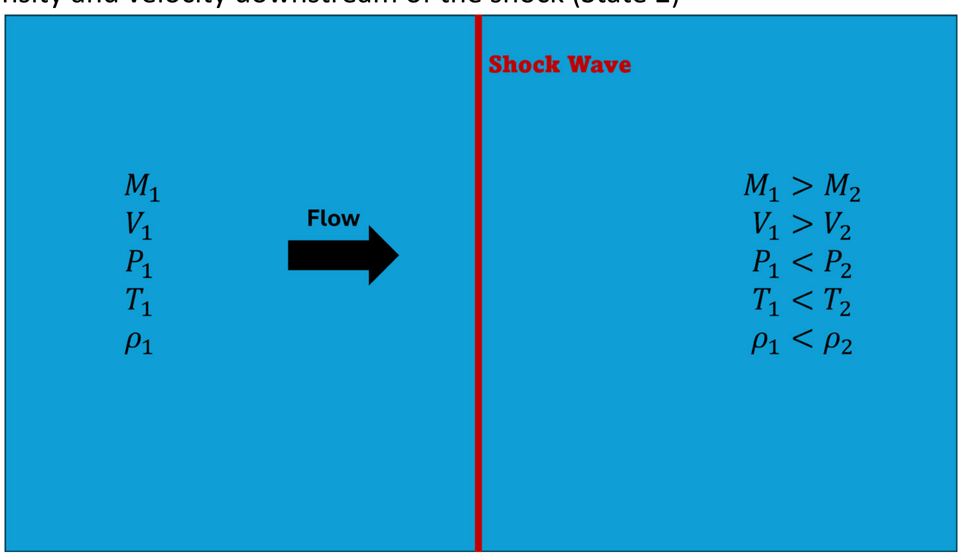

Figure 1: The Figure depicting the flow property change across the shockwave

2.2.2 Conservation of Momentum (Momentum Equation)

This principle relates the forces acting on the fluid to the change in momentum. Across a normal shock, the change in momentum flux is balanced by the pressure difference across the shock.
Understanding: The sum of the static pressure and the dynamic pressure (which represents the momentum carried by the flow) is conserved across the shock. The high-speed fluid exerts a certain

pressure and has a certain "push" due to its motion. After the shock, these components adjust but their sum remains constant.
Equation:

$$p_1 + \rho_1 u_1^2 = p_1 + \rho_2 u_2^2$$

Where:

p_1,p_2: Static pressure upstream and downstream of the shock
$\rho_1 u_1^2$,$\rho_2 u_2^2$: Momentum flux per unit area upstream and downstream

2.2.3 Conservation of Energy (Energy Equation)

This principle states that the total energy of the fluid is conserved. For an adiabatic shock (no heat added or removed from the system, and no external work done), the total energy per unit mass remains constant.
Understanding: The total energy, which includes the fluid's internal heat energy (enthalpy) and its kinetic energy (energy of motion), stays the same before and after the shock. While the forms of energy may transform (e.g., kinetic energy converting to thermal energy), the overall sum is conserved. This implies that the stagnation enthalpy (and thus stagnation temperature for an ideal gas) is constant across the shock.
Equation:

$$h_1 + \tfrac{1}{2}u_1^2 = h_2 + \tfrac{1}{2}u_2^2$$

Where:

$h_1,\ h_2$: Specific enthalpy (enthalpy per unit mass) upstream and downstream
$\frac{1}{2}u_1^2,\ \frac{1}{2}u_2^2$: Kinetic energy per unit mass upstream and downstream

2.3 Derived Relationships (Isentropic Flow ratios):

The result of combining the three conservation laws above with the ideal gas law (P=ρRT) and the definition of specific enthalpy for an ideal gas ($h = c_p T = \frac{\gamma}{\gamma-1}\frac{p}{\rho}$) .These derivations allow us to express the ratios of downstream to upstream properties solely in terms of the upstream Mach number (M_1) and the ratio of specific heats (γ).

2.3.1 Derivation of static Pressure Ratio ($^{p_2}/_{p_1}$)

To derive the pressure ratio, we combine the conservation of mass and conservation of momentum equations. We use the continuity equation to express the downstream velocity (u_2) in terms of the upstream velocity (u_1) and the densities ($\rho_1,\ \rho_2$). This expression for u_2 is then substituted into the momentum equation.
After algebraic rearrangement and introducing the Mach number (M=u/a) and the speed of sound ($a = \sqrt{\gamma p/\rho}$), the system of equations can be solved to yield the pressure ratio. This derivation is fundamental but involves several steps of substitution and simplification which we will not be discussing in this case study.
General Equation for static Pressure Ratio:

$$\frac{p_1}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

For Air (γ=1.4): Substituting γ=1.4 simplifies the equation to:

$$\frac{p_1}{p_2} = \frac{\left(7M_1^2 - 1\right)}{6}$$

What happens to pressure when you move across the normal shock?
As the fluid crosses a normal shock, its pressure significantly increases. This pressure jump is directly

related to how fast the fluid was moving initially (its upstream Mach number). The faster the incoming flow, the more intense the pressure rise across the shock. This sudden increase in pressure helps to slow down the flow.

2.3.2 Derivation of Density Ratio ($^{\rho_2}/_{\rho_1}$)

The density ratio is also derived by manipulating the conservation of mass and conservation of momentum equations, along with the definition of Mach number. Like the pressure ratio, we substitute expressions from one equation into another and use the ideal gas relations to isolate the density ratio. The process leads to an equation solely dependent on the upstream Mach number and γ.
General Equation for Density Ratio:

$$\frac{\rho_1}{\rho_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

For Air the value of γ=1.4, thus substituting γ=1.4 which simplifies the equations to:

$$\frac{\rho_2}{\rho_1} = \frac{6M_1^2}{5+M_1^2}$$

So, while moving across the normal shock, the fluid's density dramatically increases as it passes through the normal shock. This is a direct consequence of the flow slowing down: to maintain the conservation of mass flow, the fluid must become more compressed (denser) as its speed decreases. For air, there's a maximum compression limit of 6 times the original density, regardless of how fast the flow was initially.

2.3.3 Derivation of Temperature Ratio ($^{T_2}/_{T_1}$)

The temperature ratio is obtained by combining the previously derived pressure ratio and density ratio with the ideal gas law (p=ρRT). Since temperature is directly proportional to pressure and inversely proportional to density (for a gas constant R), we can simply take the product of the pressure ratio and the inverse of the density ratio. Thus, the general temperature ratio for any gas can be written as:

$$\frac{T_1}{T_2} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)\right]\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$

Thus substituting for the value of γ=1.4 corresponding to air, we get:

$$\frac{T_2}{T_1} = \frac{(7M_1^2-1)(M_1^2+5)}{36M_1^2}$$

Because of the above equation it can be clearly understood that the fluid's temperature increases significantly. This happens because a large portion of the fluid's kinetic energy (energy of motion) is irreversibly converted into internal thermal energy due to the strong dissipation within the shock layer. This heating effect is why aircraft flying at supersonic or hypersonic speeds experience extreme surface temperatures.

2.3.4 Derivation of Downstream Mach Number (M_2)

The downstream Mach number is derived by using the definitions of Mach number ($M = u/a$) and the speed of sound ($a = \gamma p/\rho$), along with the density ratio and static pressure ratio. We effectively calculate the ratio of the downstream to upstream Mach number squared, then take the square root. The process involves substituting the relations for velocity, pressure, and density ratios into the Mach number definition and simplifying. The equation then ends up becoming:

$$M_2^2 = \frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)}$$

Substituting the γ for air as 1.4, the equations simplify to:

$$M_2^2 = \frac{M_1^2+5}{7M_1^2-1}$$

If the fluid enters the shock at supersonic speed ($M_1 > 1$), it will always exit the shock at subsonic speed ($M_2 < 1$). The shock acts as a powerful diffuser, rapidly decelerating the flow. Even for extremely fast incoming flows, the downstream Mach number for air will never go below approximately 0.378 (which is $\frac{1}{\sqrt{7}}$). This fundamental change from supersonic to subsonic flow is characteristic of normal shocks.

2.3.5 Summary of Property Changes Across a Normal Shock:

The Table 1 and Figure 1 depicting the flow property change across the shockwave below depict the summary of flow properties changing across the normal shockwave according to the property equations that was described in the previous sections.

Property	**Upstream (State 1)**	**Downstream (State 2)**	**Change Across Shock**
Mach Number	Supersonic (M1>1)	Subsonic (M2<1)	Always decreases
Velocity (V)	Higher	Lower	Decreases sharply
Static Pressure (p)	Lower	Higher	Increases sharply
Static Temperature (T)	Lower	Higher	Increases sharply
Density (ρ)	Lower	Higher	Increases sharply

Table 1: Property change across the shockwave.

3. Calculating the unknowns for simulation.

Given conditions in the problem statement:

- Rin=250mm
- Re=290 mm
- L=850 mm
- Pin=250K Pa
- Tin=293 K
- Min=0.5

To determine the critical (sonic) throat area of a converging-diverging (CD) nozzle under isentropic flow of an ideal gas, we use the area-Mach number relation:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

- A is the local flow area.
- A* is the critical (throat) area where Mach number is sonic.
- γ is the specific heat ratio.
- $M = M_{in}$ is the Mach number at inlet.

Assuming γ=1.4 for air, and M_{in}=0.5 from the given conditions, we get

$$\frac{A}{A^*} = 1.34$$

In this 2D simulation setup, we consider the nozzle as having a varying radius (R) along its length.

For isentropic flow conditions, the area ratio can be translated into a radius ratio since area in 2D axisymmetric conditions scales with radius:

$$\frac{A_{in}}{A*} = \frac{R_{in}}{R*} = 1.34$$

Given this, the critical (sonic) throat radius is calculated as:

$$R* = \frac{R_{in}}{1.34} = 186.5mm$$

This defines the critical throat radius needed to achieve Mach 1 flow at the throat. Since the axial position of the throat is not defined from experimental data or specific design constraints, we assume in this study that the throat is positioned 350 mm downstream from the inlet plane of the nozzle. This provides a reasonable geometry for evaluating flow acceleration and potential shock structures in the diverging section.

Similarly, we can determine the total pressure and total temperature using the normal shock relations as outlined previously. Given the upstream pressure ratio:

$$\frac{p_{in}}{p_{o,in}} = 0.843019$$

we calculate:

$$P_{o,in} = \frac{P_{in}}{0.846019} = 296553.16Pa$$

This total pressure remains constant throughout the nozzle under isentropic flow conditions.

For the exit condition, with an area ratio:

we use iterative methods (via Python script included with the documents) to solve the area-Mach relation, obtaining:

- » Subsonic Mach number $M_{subsonic} = 0.4112$
- » Pressure ratio: $\frac{P_e}{P_{o,e}} = 0.89$

Thus:

$$P_e = 0.89 \times P_{o,e} = 0.89 \times P_{o,in} = 263932.31Pa$$

For the supersonic condition, the same process yields:

- » Supersonic Mach number: $M_{supersonic} = 1.8997$
- » Pressure ratio: $P_e / P_o = 0.149$

Thus:

$$P_e = 0.149 \times P_{o,e} = 0.149 \times P_{o,in} = 44277.98Pa$$

These subsonic and supersonic exit conditions establish the boundary parameters required for setting up the nozzle flow simulations in Ansys Fluent software.

4. Simulation Setup

In this section, we outline the simulation framework used to analyze flow through a convergent-divergent (CD) nozzle. The nozzle geometry was first created in Ansys Discovery software, using the critical throat condition derived in the previous section as a baseline design criterion. This geometry ensures that sonic conditions (Mach 1) occur at the throat.

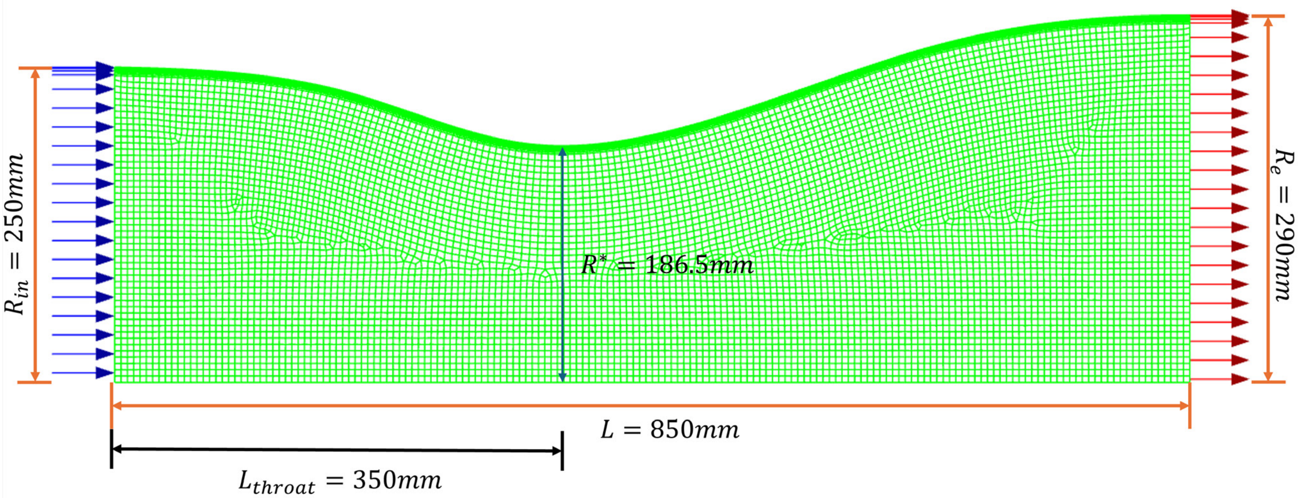

Figure 2: Convergent -Divergent Nozzle with the dimensions and the Mesh.

Following the geometry definition, computational meshing was carried out using Ansys Mechanical Software, where the fluid domain was carefully discretized into a high-quality computational grid (Figure 2).A steady-state, density-based solver in Ansys Fluent software was employed to simulate compressible flow within the nozzle. Owing to symmetry in geometry and flow, only half the nozzle domain was modeled, reducing computational effort without sacrificing fidelity. The flow was assumed to be inviscid, and the inlet boundary condition was specified as a pressure inlet with $P_{in} = 250000Pa$ and $T_{in} = 293K$, corresponding to an inlet Mach number of $M_{in}0.5$. To observe varying flow regimes, the outlet pressure was systematically varied—from a subsonic condition of $P_e = 263{,}932.31Pa$ to a supersonic condition of $P_e = 44{,}277.98Pa$—allowing the simulation to capture shock behavior and expansion phenomena across the nozzle.

5. Results

Figure 3 shows how the pressure ratio (PP0) changes along the centreline of a convergent–divergent (C-D) nozzle under different backpressure conditions. By examining this figure, we can clearly understand the three possible flow regimes in such nozzles and how normal shocks behave as operating conditions change.

5.1 Fully Supersonic Flow – Perfect Expansion

Let us first look at the yellow curve, which represents the case where the nozzle operates at the design backpressure of 44,277.97 Pa. Here, the flow accelerates smoothly from subsonic speeds at the inlet, reaches Mach 1 at the throat, and then continues expanding supersonically in the divergent section.

Notice how the pressure decreases continuously until it exactly matches the imposed backpressure at the exit. This is the ideal situation—called perfect expansion—and importantly, no shock waves appear inside the nozzle.

5.2 Sub-Critical Operation – Shock Formation Inside the Nozzle

When we increase the backpressure above this design value, the flow cannot remain supersonic all the way to the exit. Instead, the flow needs to suddenly compress in order to match the higher downstream pressure. This sudden compression happens through a normal shock wave inside the nozzle.

In Figure 3, this appears as a sharp, vertical rise in the pressure ratio curve. For backpressures of 125000 Pa, 150000 Pa, 175000 Pa, 200000 Pa, and 225000 Pa, you can clearly see that the shock position shifts upstream as the backpressure increases. At 125000 Pa, the shock is located deeper in

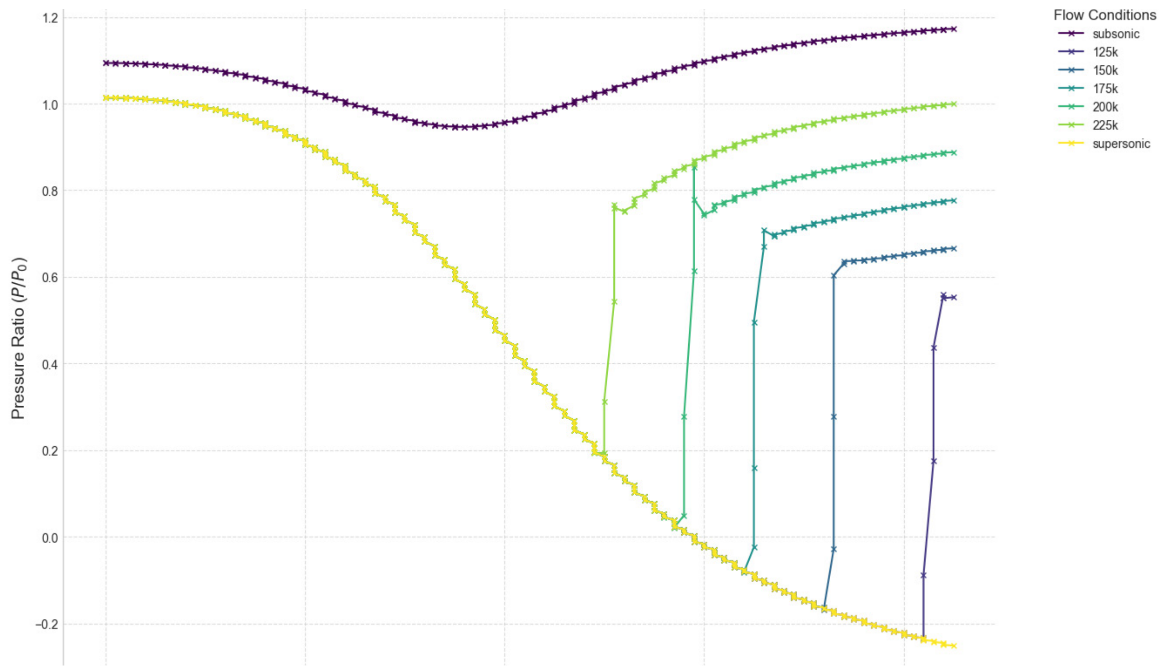

Figure 3 : The Graph depicts the Pressure ratio change at the center line of a CD Nozzle for various Back pressure values.

the divergent section and as we increase to 150000-225000 Pa, the shock gradually moves closer to the throat. Eventually, if the backpressure is raised enough, the shock sits exactly at the throat. This trend is important: it shows that the higher the backpressure, the earlier the shock must occur to provide the required pressure rise before the flow exits the nozzle.

Fully Subsonic Flow

At even higher backpressures, the flow is no longer able to accelerate to supersonic speeds at all. The entire nozzle then operates in a subsonic regime, behaving much like a simple venturi. The pressure decreases smoothly toward the throat and then increases again through the diverging section. In this case, no shock forms, because the flow never reaches supersonic speeds in the first place.

The provided temperature, density, and pressure contour plots for the Convergent-Divergent (CD) nozzle effectively visualize the theoretical behavior of a normal shock wave. The series of plots from 'a' to 'f' demonstrates the effect of increasing back pressure on the location and characteristics of the shock. This dynamic is a fundamental concept in compressible fluid dynamics, specifically for a choked nozzle operating in the over-expanded pressure regime. The plots provide a compelling graphical representation of the non-isentropic flow changes predicted by the Rankine-Hugoniot relations.

5.3 Contour Plots

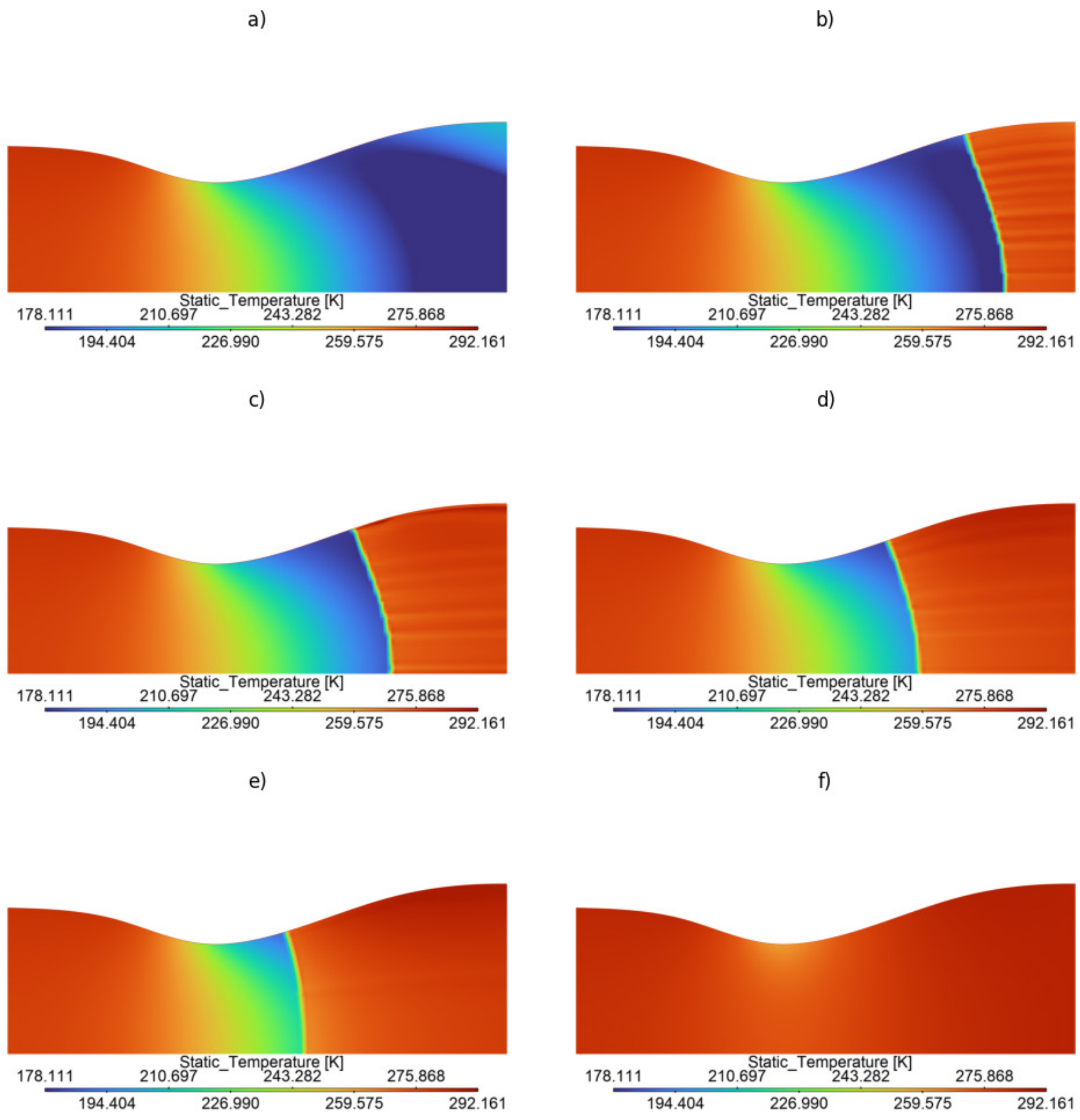

Figure 4 : The Temperature Contour plots across CD Nozzle at a) Supersonic, b) 150000 Pa c) 175000 Pa, d) 200000 Pa, e) 225000 Pa, f) Subsonic Back Pressure conditions

While the centerline pressure plots in Figure 3 provide quantitative evidence of shock behavior, the contour plots of temperature, density, and pressure (Figure 4, Figure 5, Figure 6 respectively generated using Ansys Ensight Software) give us a more intuitive, visual understanding of what is happening inside the nozzle. Each set of plots corresponds to a different backpressure condition, allowing us to track how the shock develops and moves as operating conditions change.

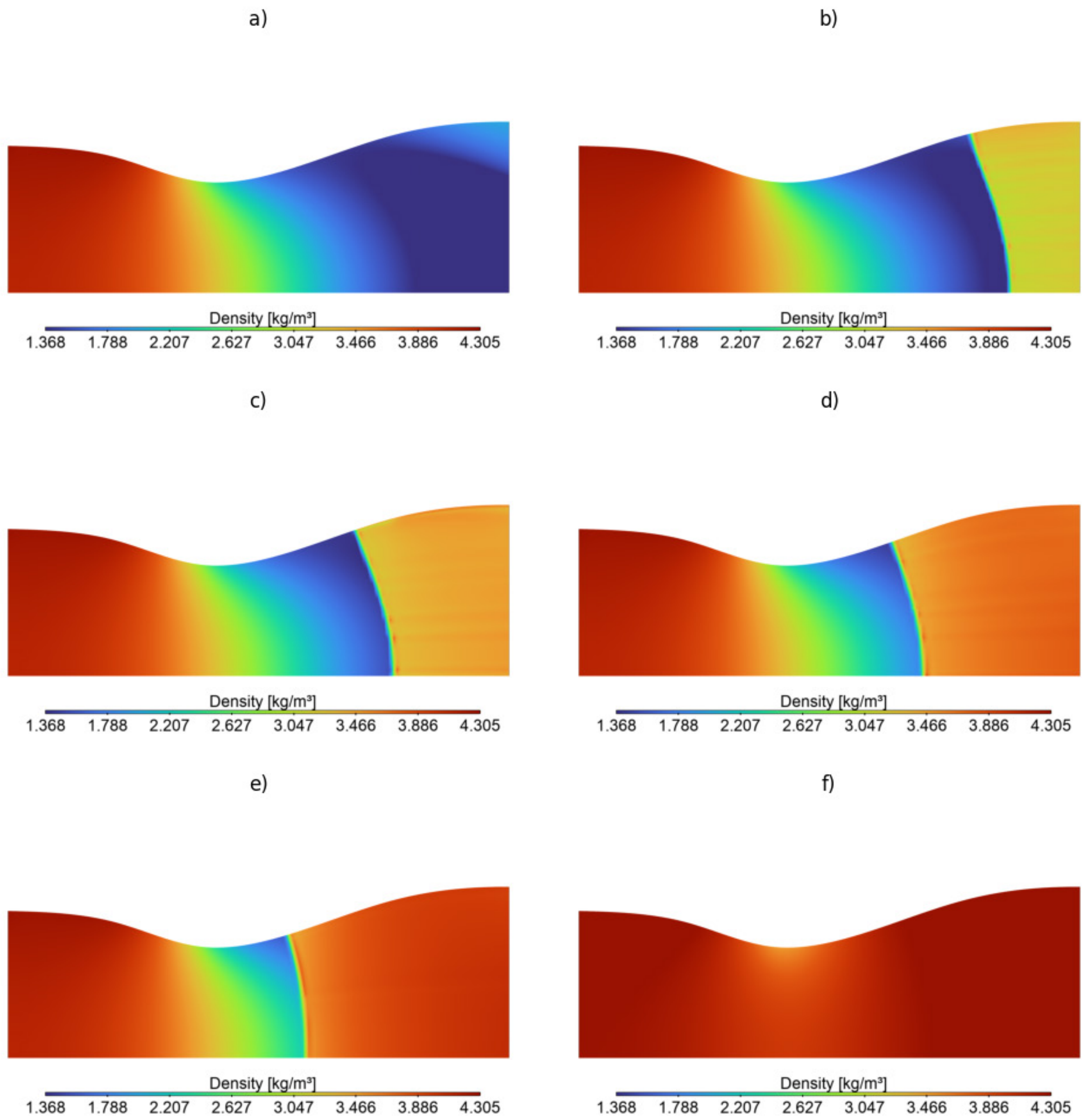

Figure 5 : The Density Contour plots across CD Nozzle at a) Supersonic, b) 150000 Pa c) 175000 Pa, d) 200000 Pa, e) 225000 Pa, f) Subsonic Back Pressure conditions

In the temperature plots (Figure 4), the shock is visible as a thin, nearly vertical band where the color transitions abruptly from cool tones (blue/green) upstream to warm tones (orange/red) downstream. This jump indicates a sudden increase in static temperature across the shock. Physically, this occurs because the supersonic flow upstream of the shock has high kinetic energy but relatively low thermal energy. When the shock forms, part of this kinetic energy is irreversibly converted into internal energy, raising the temperature sharply. As backpressure increases, we see this “temperature wall” shifting upstream, confirming the shock’s movement toward the throat.

The density plots (Figure 5) display a similar story. Before the shock, the supersonic expansion produces very low-density flow. At the shock location, there is an abrupt jump to much higher density, consistent with the compressive nature of a shock wave. Importantly, the density change is even more visually striking than temperature because shocks are fundamentally compression waves. Tracking the density contours across Figures a–f provides a very clear indication of how the shock shifts position under increasing backpressure.

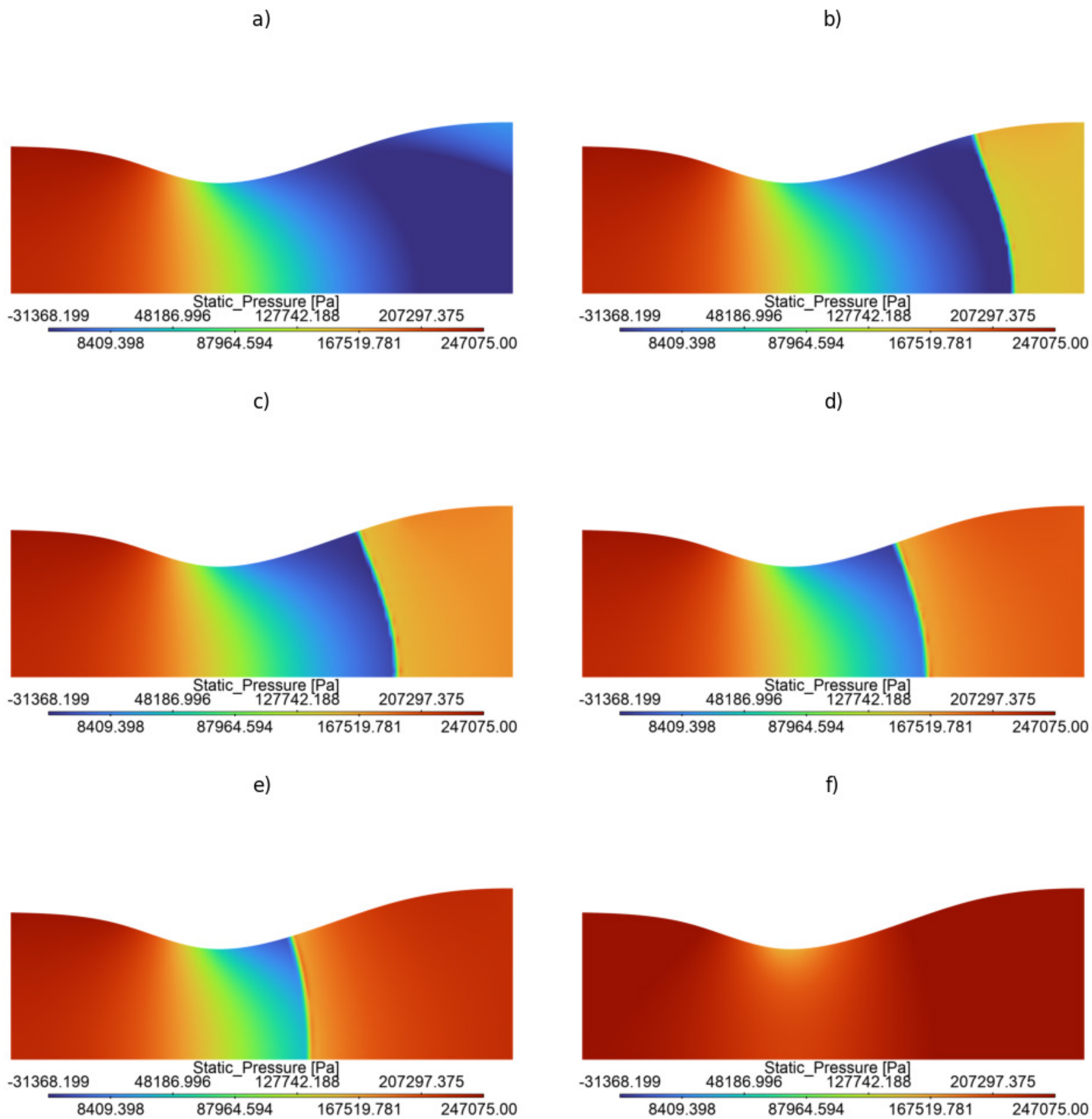

Figure 6 : The Pressure Contour plots across CD Nozzle at a) Supersonic, b) 150000 Pa c) 175000 Pa, d) 200000 Pa, e) 225000 Pa, f) Subsonic Back Pressure conditions

The pressure plots (Figure 6) provide the clearest and most direct confirmation of shock dynamics. Upstream of the shock, the pressure is very low due to expansion. At the shock front, we see a sudden increase to much higher-pressure levels (yellow/red). Downstream of the shock, the pressure continues to gradually rise toward the nozzle exit, eventually reaching the imposed backpressure.

By comparing conditions from 150 kPa up to 250 kPa, we see how the location of this sharp pressure rise progressively moves upstream with increasing backpressure. This demonstrates the nozzle's need to recompress earlier when the exit pressure requirement is higher.

6. Conclusions

Through this Problem-Based Learning (PBL) exercise, the flow characteristics of a 2D convergent–divergent nozzle have been systematically investigated, with particular emphasis on the role of normal shock waves in determining nozzle performance. Beginning with an inlet Mach number of 0.5 and an inlet pressure of 250 kPa, the analysis first established the critical throat radius required for choked flow, confirming that Mach 1 is reached at the throat under design conditions. This provided the foundation for exploring both subsonic and supersonic exit flow regimes.

The results showed that when the nozzle operates at the design backpressure, the exit Mach number matches the isentropic prediction, and the flow expands smoothly without shocks. However, as the backpressure increases, the flow undergoes a sharp recompression through a normal shock wave located within the divergent section. The exact position of this shock was shown to depend on the imposed backpressure: at moderate over-expansion, the shock forms deep in the divergent region, while at higher backpressures it migrates progressively upstream, eventually stabilizing at the throat. Beyond this point, the flow transitions to a fully subsonic regime where the nozzle behaves like a venturi.

Here, Ansys Fluent software played a crucial role by enabling the visualization of shock formation and migration. While isentropic and shock theory provide predictive equations, the Fluent-generated contour plots of pressure, temperature, and density allowed the abrupt changes across the shock to be clearly observed. The simulation revealed how kinetic energy is converted into internal energy downstream of the shock, seen as sharp jumps in temperature and density, and how static pressure rises steeply across the shock. By progressively adjusting backpressure within Ansys Fluent software, the upstream movement of the shock was visualized in real-time, making the theoretical trends far more tangible for learners.

From a learning perspective, this case study reinforces three critical insights:

- Choked flow fixes the throat Mach number at unity, controlling the mass flow rate.
- Shock position is governed by backpressure, not geometry alone, highlighting nozzle sensitivity to boundary conditions.
- Simulation complements theory—Ansys Fluent visualization transforms abstract equations into visual, interpretable flow fields, making the physics of shocks easier to grasp and more engaging to study.

Overall, this exercise demonstrates how combining gas dynamics theory with CFD visualization in Ansys Fluent software not only validates the predictions of normal shock theory but also provides students with a richer, more intuitive understanding of compressible flow phenomena.

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