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Study of Nonlinear Parametric Response in a Beam using ANSYS

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Presentation Overview

- Parametric Response Phenomenon
- Modeling Assumptions in ANSYS
- Simulations Assuming Proportional Damping
- Quadratic Damping Implementation
- Simulations Assuming Quadratic Damping
- Phase Between Input and Response
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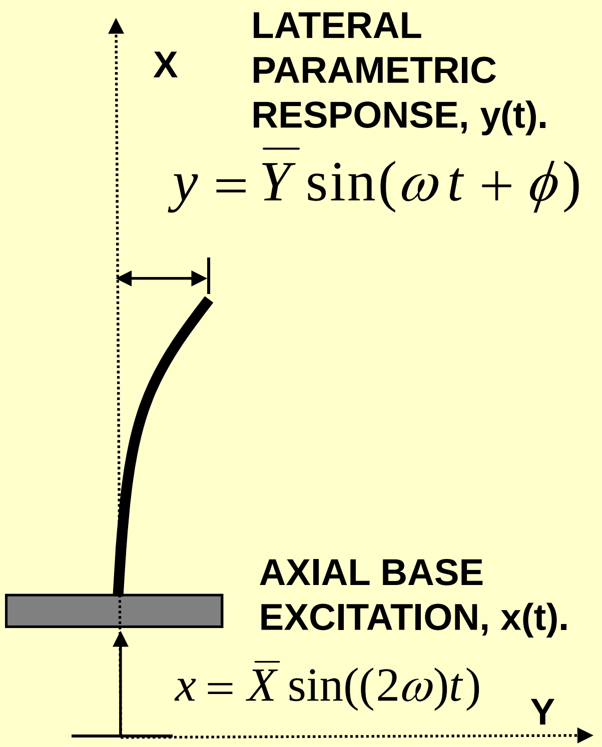
Parametric Response Phenomenon

- Nonlinear parametric vibrations can occur in flexible structures when an excitation frequency is close to twice one of the natural frequencies of the system .
- A practical example of a structure that could experience parametric response is a cable-stayed bridge cable. Excitation near 2X a natural frequency due to bridge deck motion could cause large amplitude cable response at half the excitation frequency.

Parametric Response Phenomenon

• Several researchers have considered a flexible cantilever beam as a simple and convenient structure to study parametric response.

• A flexible beam with axial base excitation at a frequency that is 2X a natural frequency can exhibit parametric response in the lateral direction.



Parametric Response Phenomenon

- T.J. Anderson conducted studies for parametric resonance of a flexible beam:
  - Anderson, T.J (1993), Nonlinear Vibrations of Metallic and Composite Structures, PhD Dissertation, Virginia Polytechnic Institute and State University.
- This presentation describes transient simulations of parametric response of the first mode of Anderson's beam. Additional references and details can be found in:
  - Remala, S.N.R (2005), Nonlinear Transient Finite Element Simulations of Beam Parametric Response Including Quadratic Damping, M.S. Thesis, University of Kentucky.

Modeling Assumptions in ANSYS

- Anderson's flexible cantilever beam:

Young's Modulus	30 x10^6 lb/in^2
Density	0.00073 lb.s^2
Possion's Ratio	0.29

Length *(L)*	33.56 in
Height *(h)*	0.032in
Width *(w)*	0.75 in

- The beam's first natural frequency for lateral motion, including gravity loading, is ω_n=0.637 Hz.
- Imposed axial displacement is $x = \overline{X}\sin(2\omega t)$, where $\omega \approx \omega_n$.
- Amplitude of the base input displacement is adjusted for each input frequency to maintain an acceleration amplitude of 46.53 in/s^2.
- Initial condition is a transverse velocity, v_o, of 0.5 in/s^2 at the free end.

Modeling Assumptions in ANSYS

- ANSYS Beam3 2D beam element is used.
- A transient simulation is run:
  - *antype,trans*
  - *trnopt,full*
- Large deflection nonlinearities are included:
  - *nlgeom,on*
- Stress stiffening is included:
  - *sstif,on*
- Gravity loading is included:
  - *acel,386.4*

Modeling Assumptions in ANSYS

- The axial displacement input is imposed using a table named “disp”:

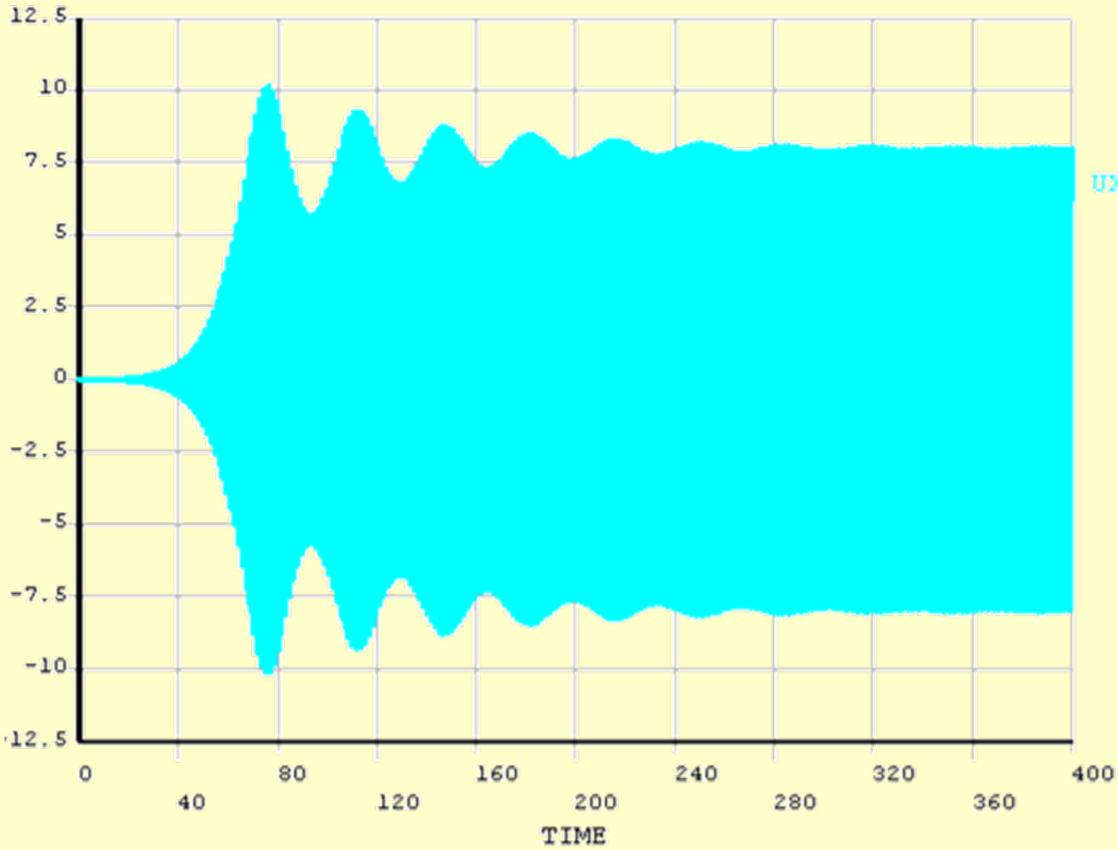
```
f=1.26
a=46.53/((2*3.14159265359*f)**2)
*do,i,1,100001
        t=(i-1)/1000
        disp(i,1)=a*sin(2*3.14159265359*f*t)
        disp(i,0)=t
*enddo
d,1,uy,0
d,1,rotz,0
d,1,ux,%disp%
```

Simulations Assuming Proportional Damping

- Initially, simple proportional damping was used. .
- The lateral-direction free-end response is shown for time from 0 to 400 seconds assuming simple proportional damping:

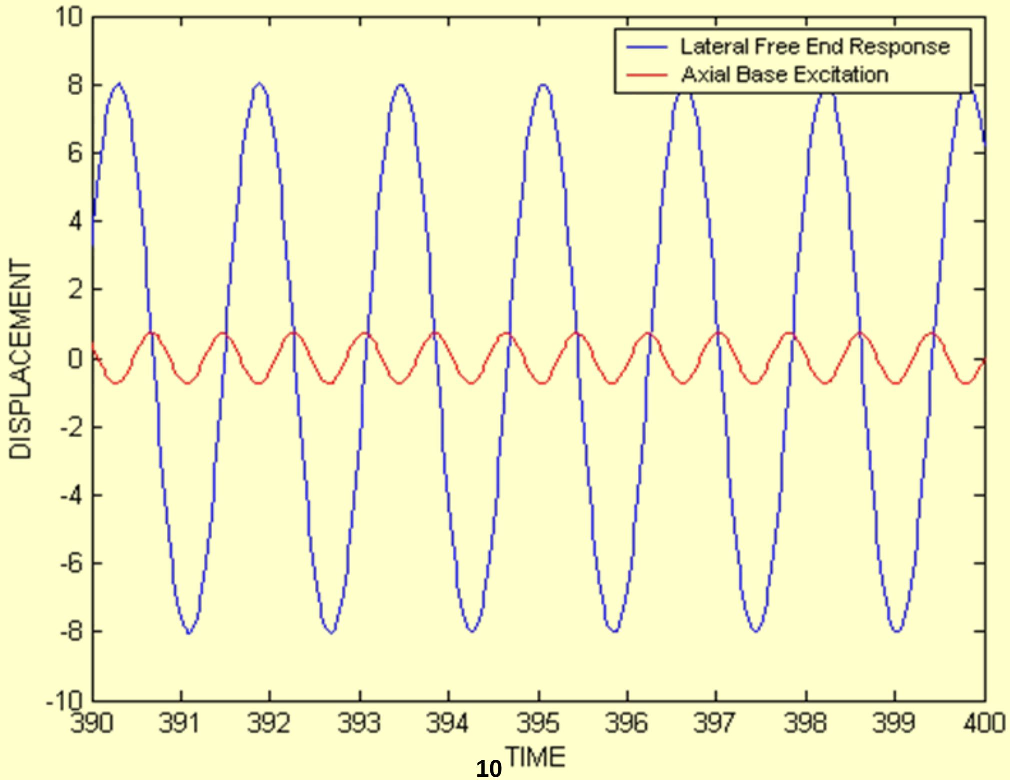
  alphad, 0.0015

  betad, 0.0015.



Simulations Assuming Proportional Damping

- For a time range of 390-400 seconds, there are about 12.6 cycles of axial displacement input (red) and about 6.5 cycles of lateral free end response (blue)
  - Response is parametric; steady state is clearly reached



Quadratic Damping Implementation

- Anderson concluded that inclusion of quadratic damping with linear structural damping improved agreement between experimental and theoretical results for beam parametric vibration
- Quadratic damping was implemented, along with linear structural damping, in ANSYS simulations
- There is a user-option in ANSYS to implement quadratic damping using Combin14 elements as dashpots
  - However, a different, more flexible approach, is implemented in this work using the ANSYS parametric design language (APDL)

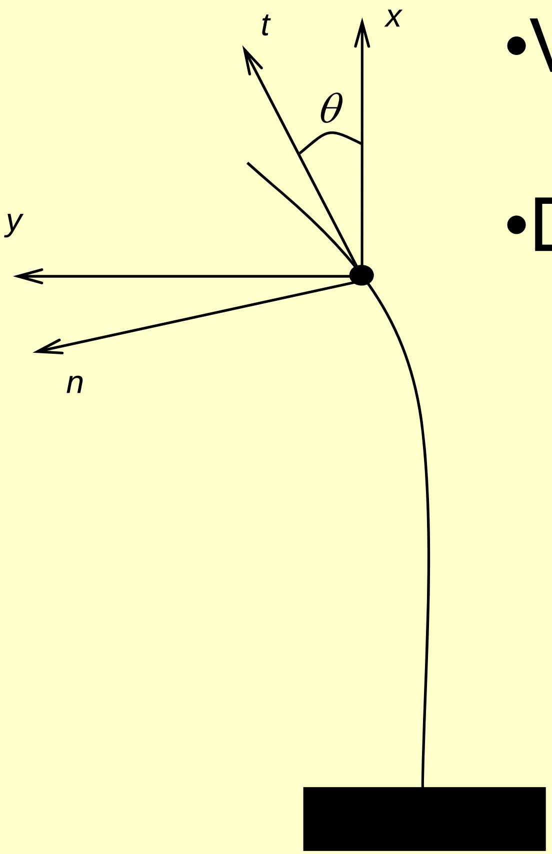
Quadratic Damping Implementation

- This method allows for the damping force to remain normal to the beam for large deflections
- A transient dynamic analysis is performed for a small interval of time, and the ANSYS **get* command is used to retrieve nodal displacements and rotations after the small interval of time
  - Velocity components in the x and y-directions are calculated
- The corresponding normal and tangential velocity components are calculated based on the x and y velocity components at each node
- Damping force components, proportional to the squares of the velocities, are calculated at each node

Quadratic Damping Implementation

- The damping forces are applied as structural forces in a subsequent, short-duration transient simulation
- The process is repeated for as many time intervals as required until the response reaches steady-state
- A do-loop is implemented using APDL to perform repeated transient simulations
- The nodal quadratic damping force calculation is summarized on the following slide

Quadratic Damping Implementation



- Velocity components at a node:

v_t, v_n, v_x, v_y

- Damping force at a node: f_d

$$\begin{bmatrix} v_t \\ v_n \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

$$f_d = -Dv_n |v_n|$$

$$\begin{bmatrix} (f_d)_x \\ (f_d)_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ f_d \end{bmatrix}$$

n – Normal direction at a node

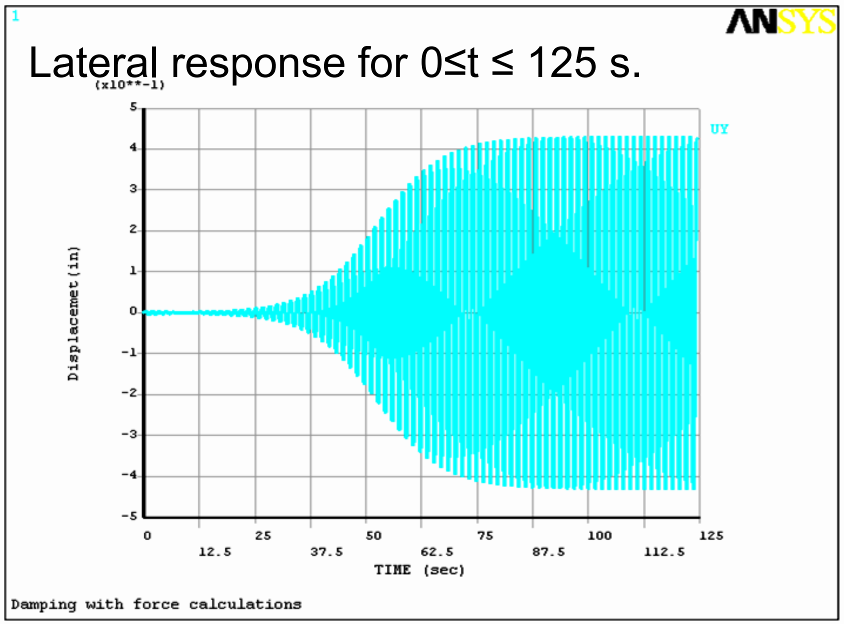
t – Tangential direction at a node

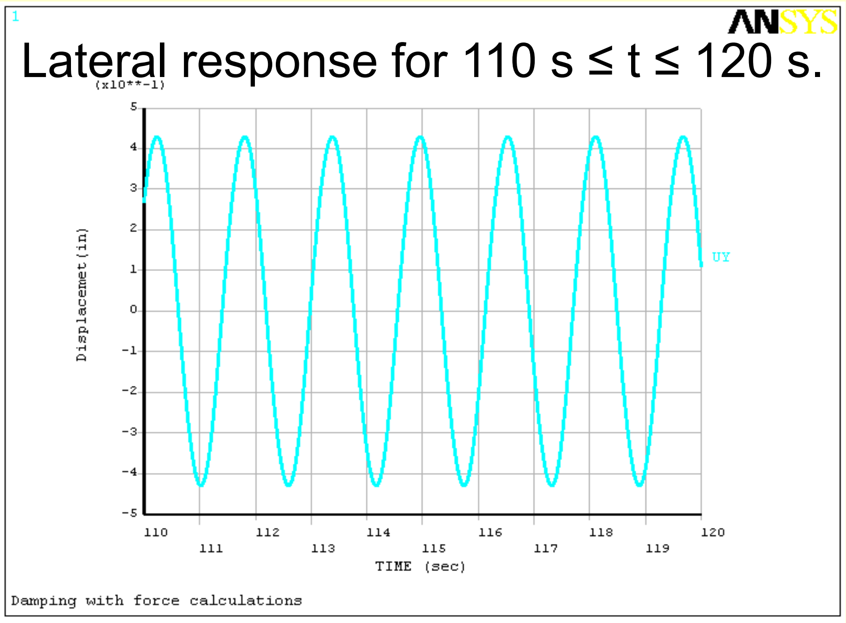
$$(f_d)_x = (-\sin\theta) f_d$$

$$(f_d)_y = (\cos\theta) f_d$$

Simulations Assuming Quadratic Damping

- Results using quadratic damping are shown with an axial base displacement excitation at f_{ex}=1.27 Hz. Response is at approximately at 0.63 Hz, near the first natural frequency (about half the excitation frequency).





Simulations Assuming Quadratic Damping

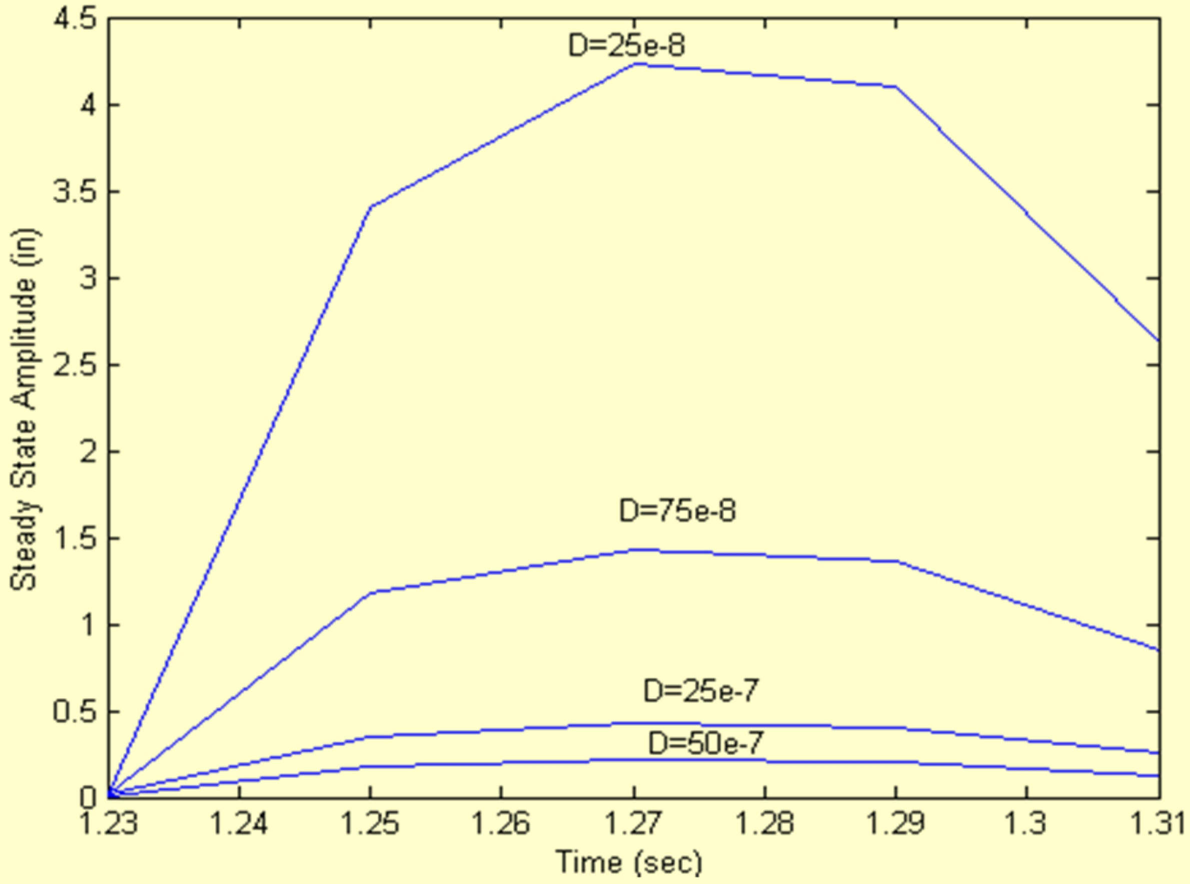
- Using the quadratic damping assumption, transient simulations were performed for a range of excitation frequencies (f_{ex} = 1.21, 1.23, 1.25, 1.27, 1.29, 1.31, 1.33 Hz) and quadratic damping coefficients (D= 25E-8, 75E-8, 25E-7, and 50E-7 lb-s^2/in^2).

- No parametric response was predicted at the extremes of the range (f_{ex} = 1.21 Hz, f_{ex} = 1.33 Hz).

- Parametric response was predicted for the excitation frequency range: f_{ex} = 1.23 Hz - 1.31 Hz.

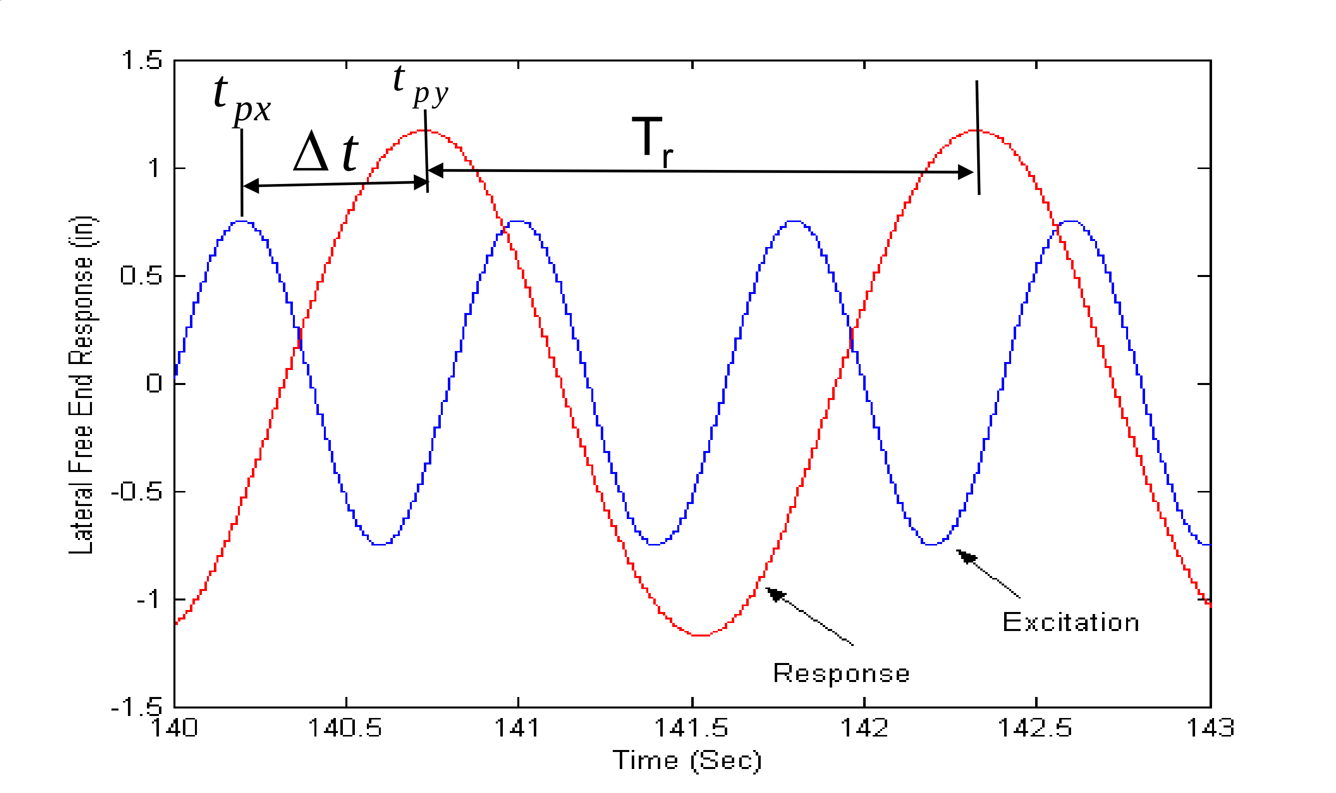
Simulations Assuming Quadratic Damping

- Steady-State Response Amplitude vs. Excitation Frequency:



Phase Between Input and Response

- Response phase is not clearly defined in parametric response case. Here, we define it based on the following figure and the discussion on the following slide. It is of interest in understanding parametric response.



Phase Between Input and Response

- Phase Difference defined as: $\phi = \pi(\Delta t)/T_r$
- Response phase (in degrees) from ANSYS simulations as a function of excitation frequency, f_{ex}, and quadratic damping coefficient, D, is shown in the table below:

f_{ex} (Hz)	D=25E-8 (lb-s^2/in^2)	D=75E-8 (lb-s^2/in^2)	D=25E-7 (lb-s^2/in^2)	D=75E-7 (lb-s^2/in^2)
1.23	46.7	47.8	47.8	47.4
1.25	58.3	60.0	59.3	59.6
1.27	66.0	68.9	66.1	67.6
1.29	73.1	74.8	74.8	73.8
1.31	81.7	82.9	80.0	82.9

Summary

- Implementation of quadratic damping in analysis of beam parametric response using ANSYS was presented
  - A study of response amplitude as a function of quadratic damping coefficient, D, and excitation frequency, f_{ex} was conducted
- The response phase is of interest in understanding the nonlinear parametric response phenomenon
  - It was calculated for a range of quadratic damping coefficients and excitation frequencies
- It was observed that, as defined here, the phase varies roughly from 45^o to 90^o in the range of excitation frequencies for which parametric response is predicted
  - The phase did not appear to depend on the damping coefficient