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Simulation of Densification in Granular Materials and Geomaterials

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Overview

- An implicit formulation of a three-invariant, isotropic / kinematic hardening Cap plasticity model was programmed into ANSYS **V10** as a user material subroutine (usermat3d.f)
  - Based on Foster, et al. (2005) with corrections and a few mods.
- Used to simulate behavior of rocks, ceramics, sand, soil, powders, etc.
- Examples:
  - Single element tests
  - Simulated multi-element hydrostatic & triaxial test samples

Brief Review of Some Terms

- Stress Invariants
  - I_1 : (measure of hydrostatic pressure)

    $$I_1 = -3p = \tau_{KK}$$
  - J_2: (measure of shear stress)

    $$J_2 = \frac{1}{2}\tau'_{ij}\tau'_{ij}$$
  - J_3: (incorporating difference strengths in [triaxial] extension and [triaxial] compaction.)

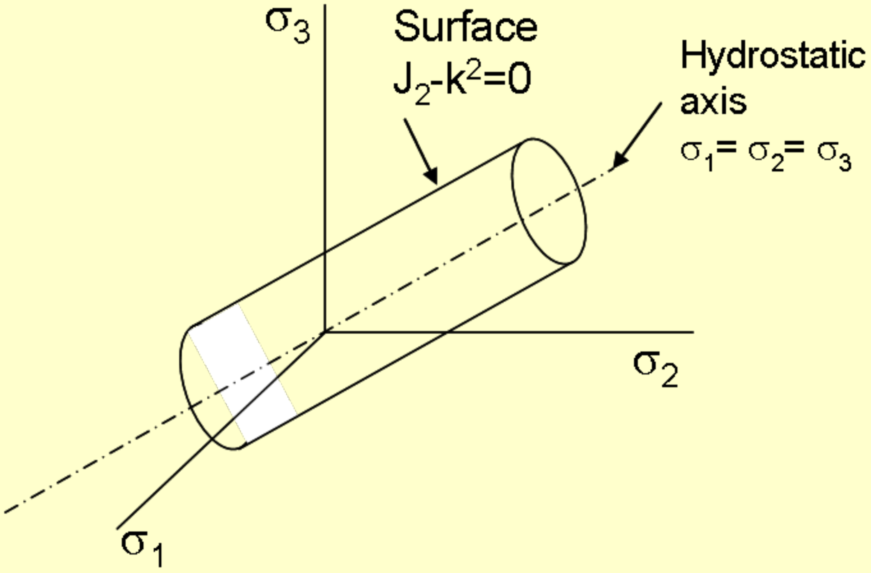
    $$J_3 = \frac{1}{2}\tau'_{ij}\tau'_{jk}\tau'_{ki}$$

    $$\tau'_{ij} = \tau_{ij} - \delta_{ij}\frac{\tau_{KK}}{3}$$
- Relative Stress, ξ_{ij}

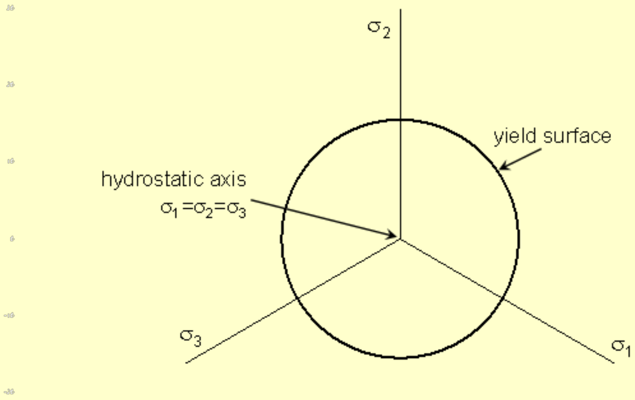
  $$\xi_{ij} = \tau_{ij} - \alpha_{ij}$$
- Back Stress, α_{ij}
- Lode angle, β

  $$\cos(3\theta) = -\sin(3\beta) = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$$

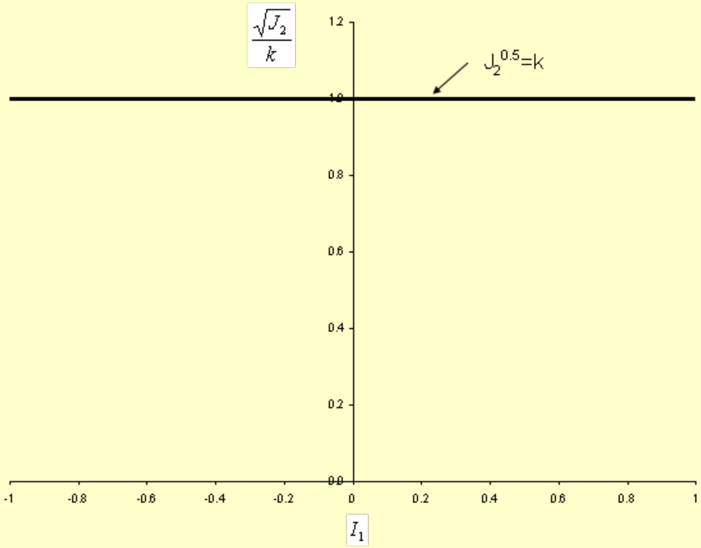
Von Mises Yield Criteria



principal stress space

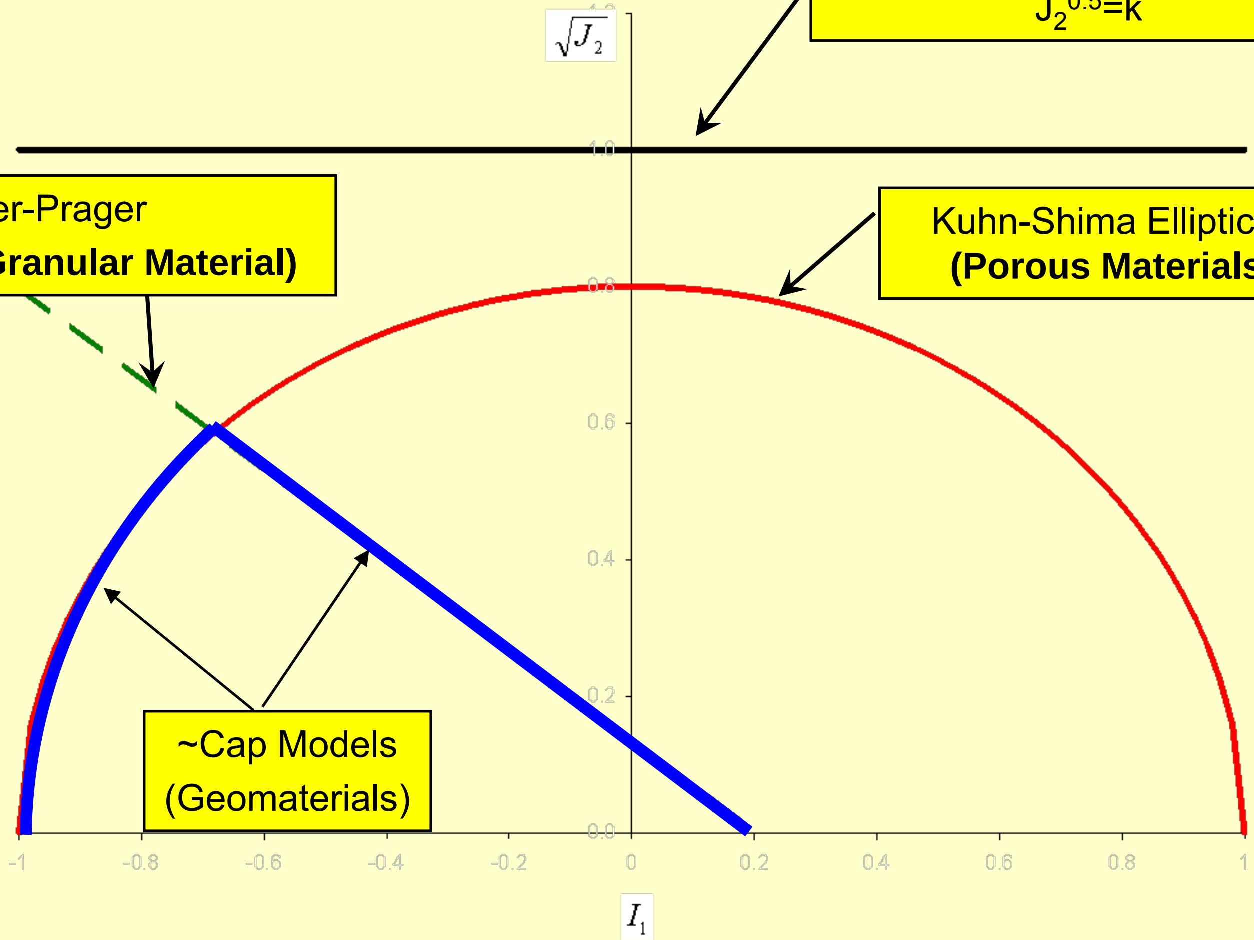


π plane projection



I_1-$J_2^{0.5}$ space

Various Yield Criteria in I_1-$J_2^{0.5}$



VonMises Criteria (**Metals**)
$J_2^{0.5}=k$

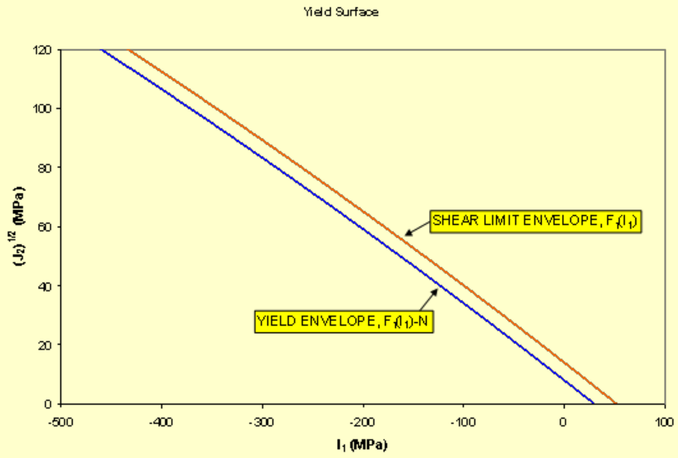
Drucker-Prager
(Frictional or Granular Material)

Kuhn-Shima Elliptical
(Porous Materials)

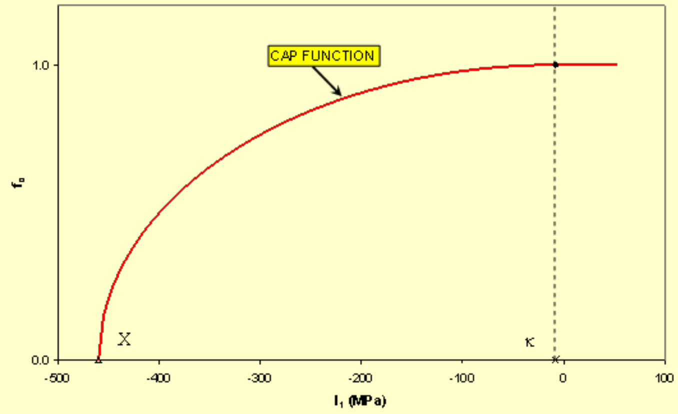
~Cap Models
(Geomaterials)

Continuous Cap Yield Surface

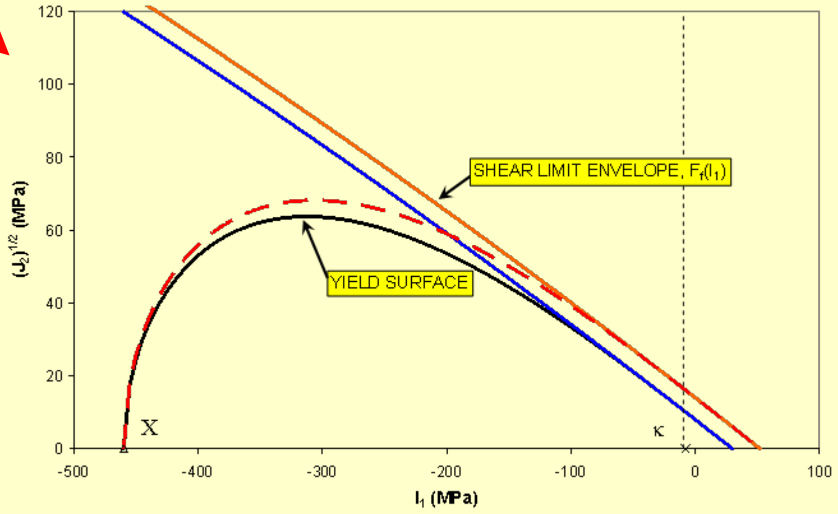
Shear Failure Surface (~ Drucker-Prager)



Cap Function



Continuous Cap Yield Surface

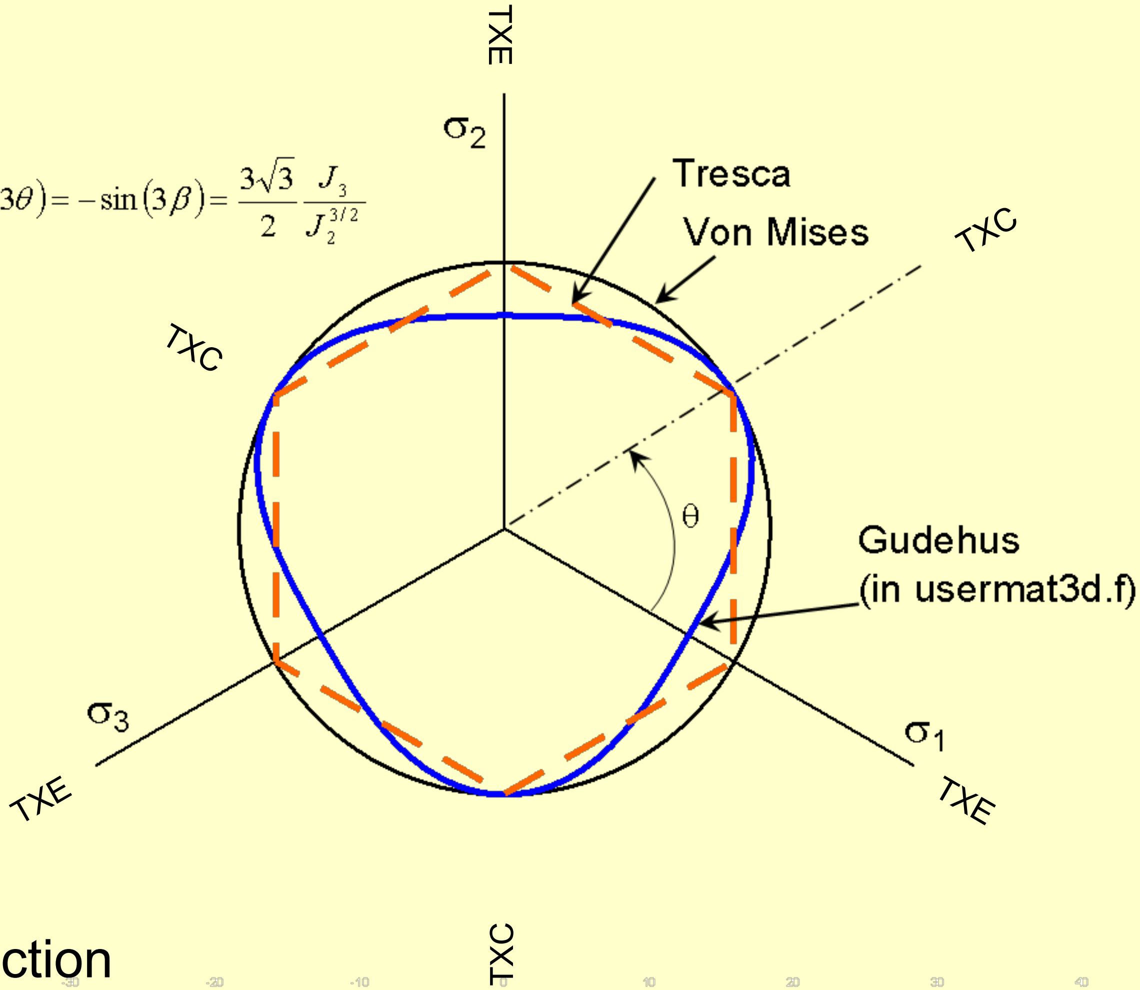


Cap moves with κ

κ based on ε_v^p

J_3 Dependence (π Plane)

$$\cos(3\theta) = -\sin(3\beta) = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$$



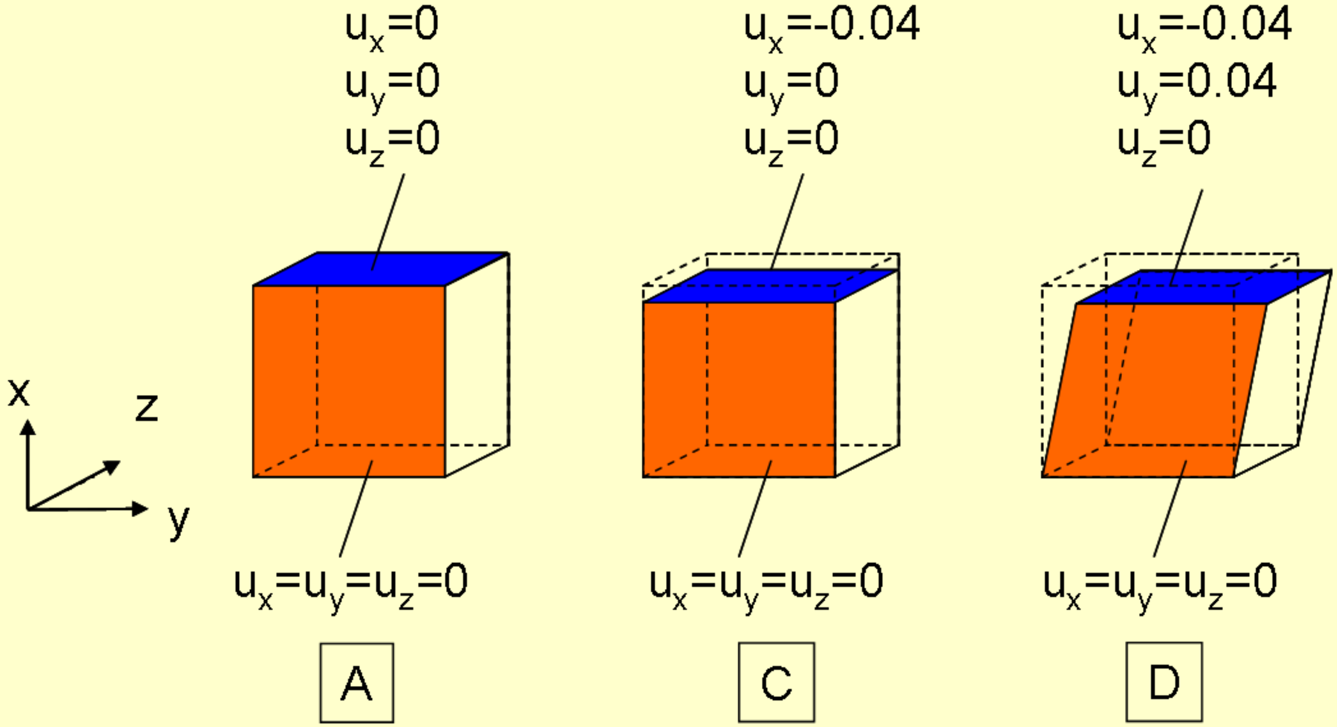
π plane projection

Description of Model (usermat3d.f in ANSYS Ver.10)

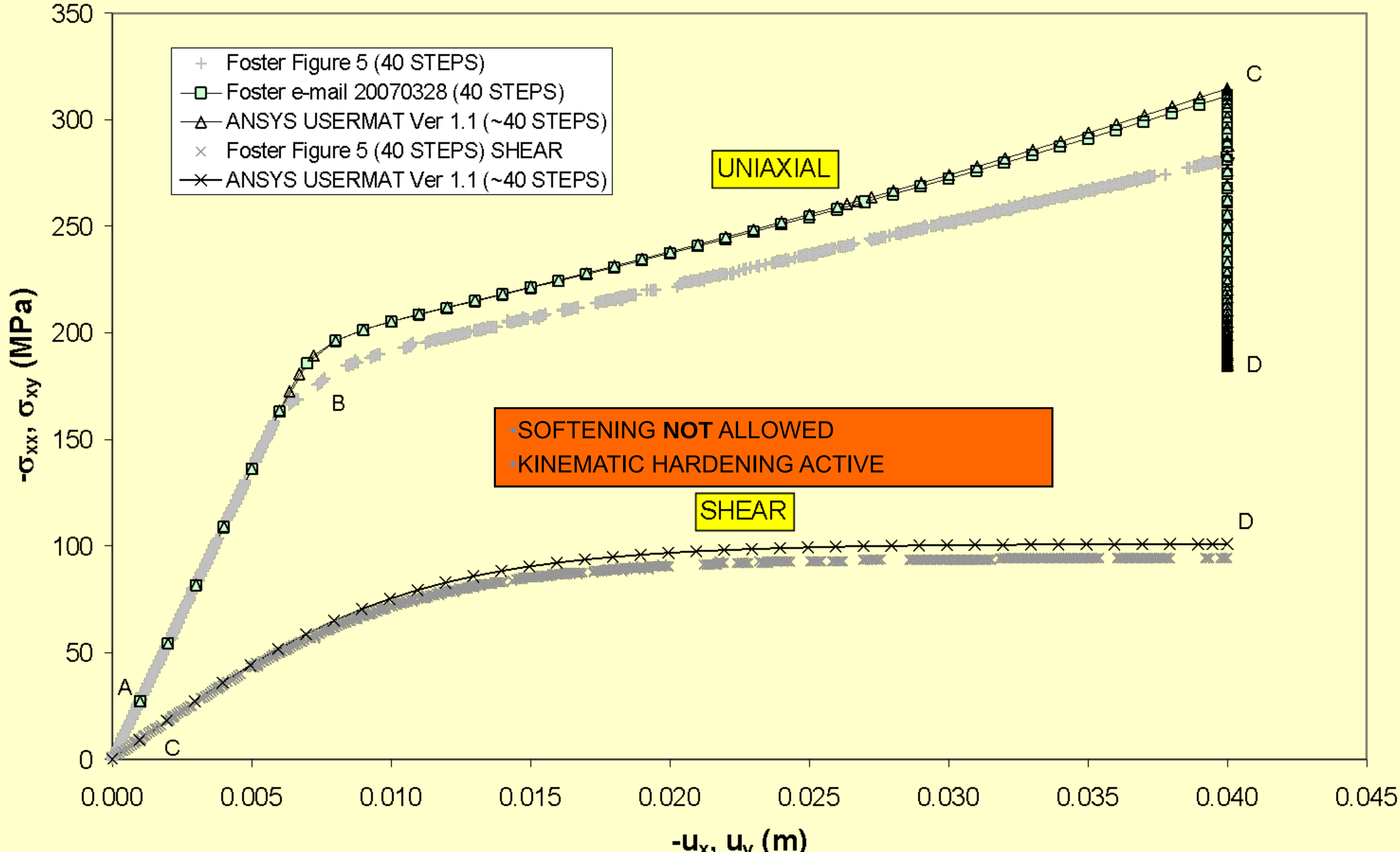
- Based on Foster, et al (2005)
  - Corrections
  - Modifications
    - Switch to prevent cap softening
      - OFF (Allowed): Soils, etc. (i.e. no restriction on $\Delta\kappa$)
      - ON (Prevented): Rocks, concrete, etc. (i.e. $\Delta\kappa \leq 0$)
    - Variable elastic properties (table and ON/OFF switch)
      - $E=f(\rho_\psi)$, $\nu=g(\rho_\psi)$
      - Constant during a time step
    - Densities tracked
      - $\rho_\psi=\rho_i \exp(-\varepsilon_V^{PL})$ "free state density"
      - $\rho=\rho_i \exp(-\varepsilon_V)$ density

Single Element Results (Example #1)

- Plane strain compression then shear.
- Constants from Box 2 of Foster, et al. (2005)
  - Salem Limestone

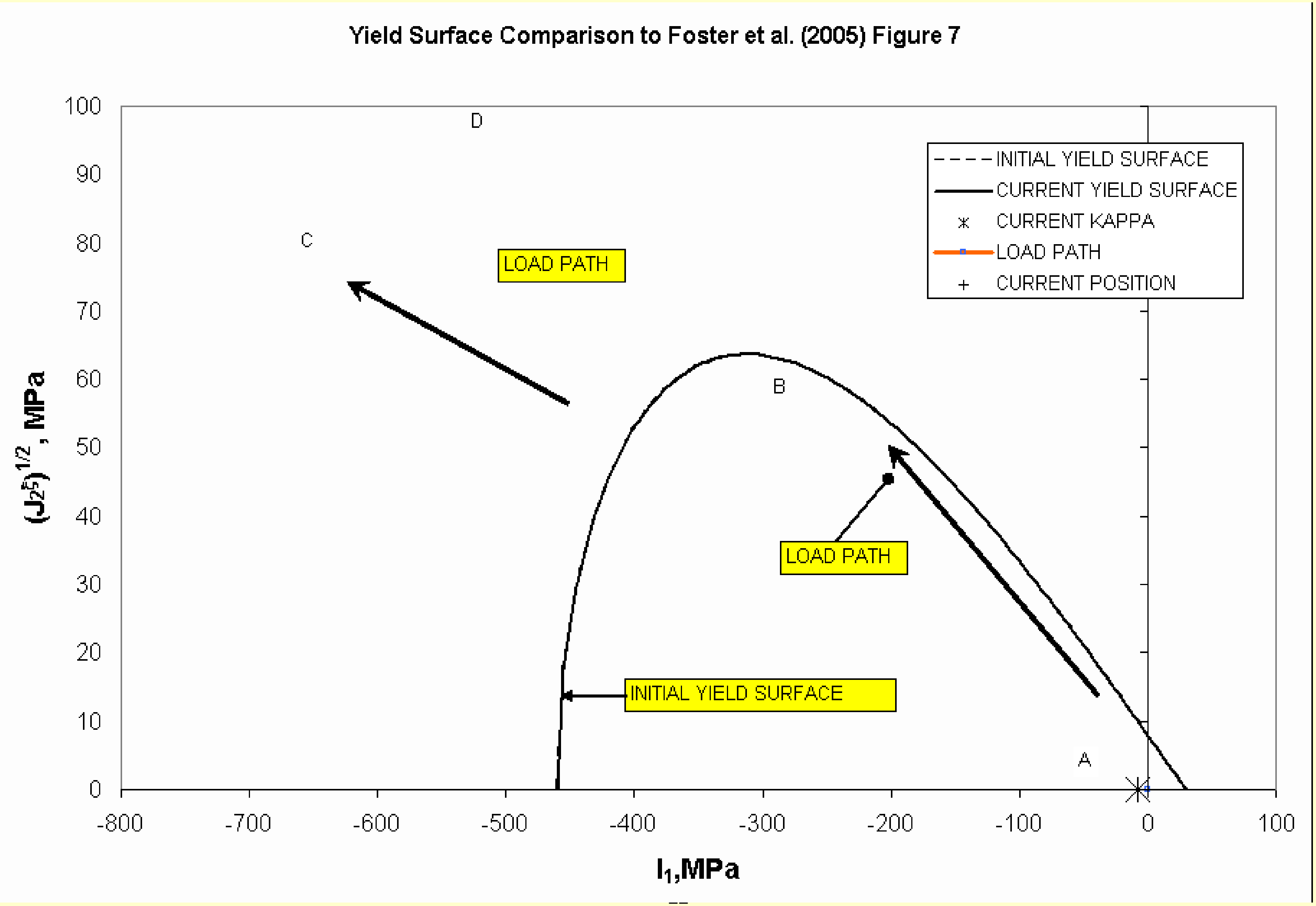


Comparison with Foster, et al. (2005) (Example #1)



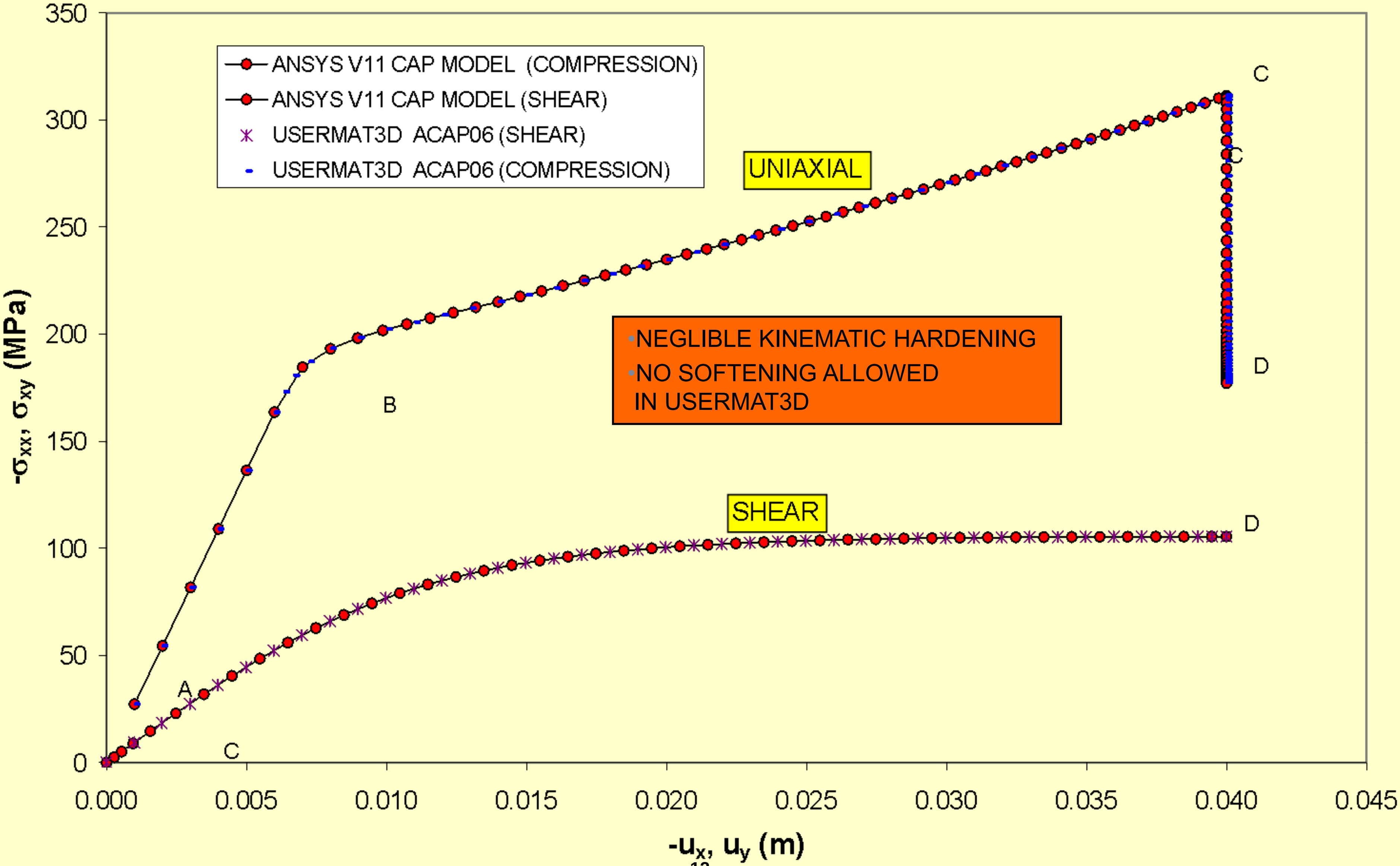
- SOFTENING **NOT** ALLOWED
- KINEMATIC HARDENING ACTIVE

Comparison with Foster, et al. (2005) (Example #1)



Yield Surface Comparison to Foster et al. (2005) Figure 7

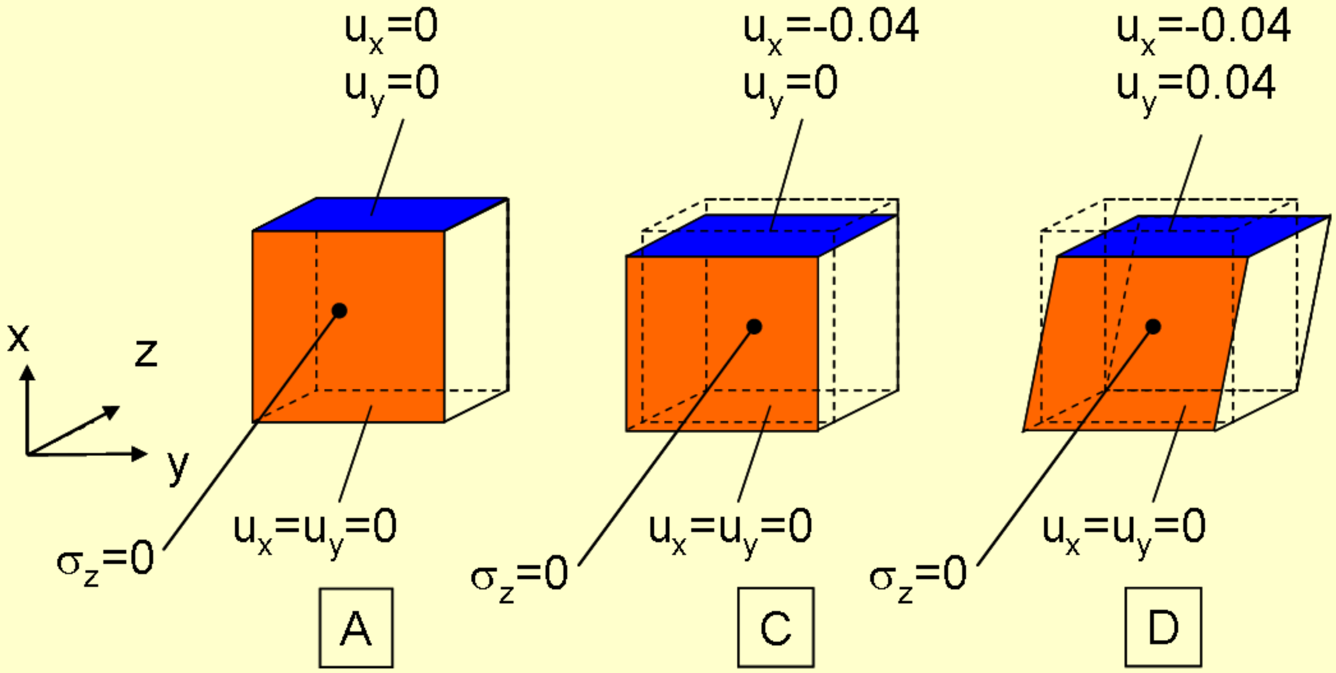
Comparison with ANSYS V11 CAP (Example #1)



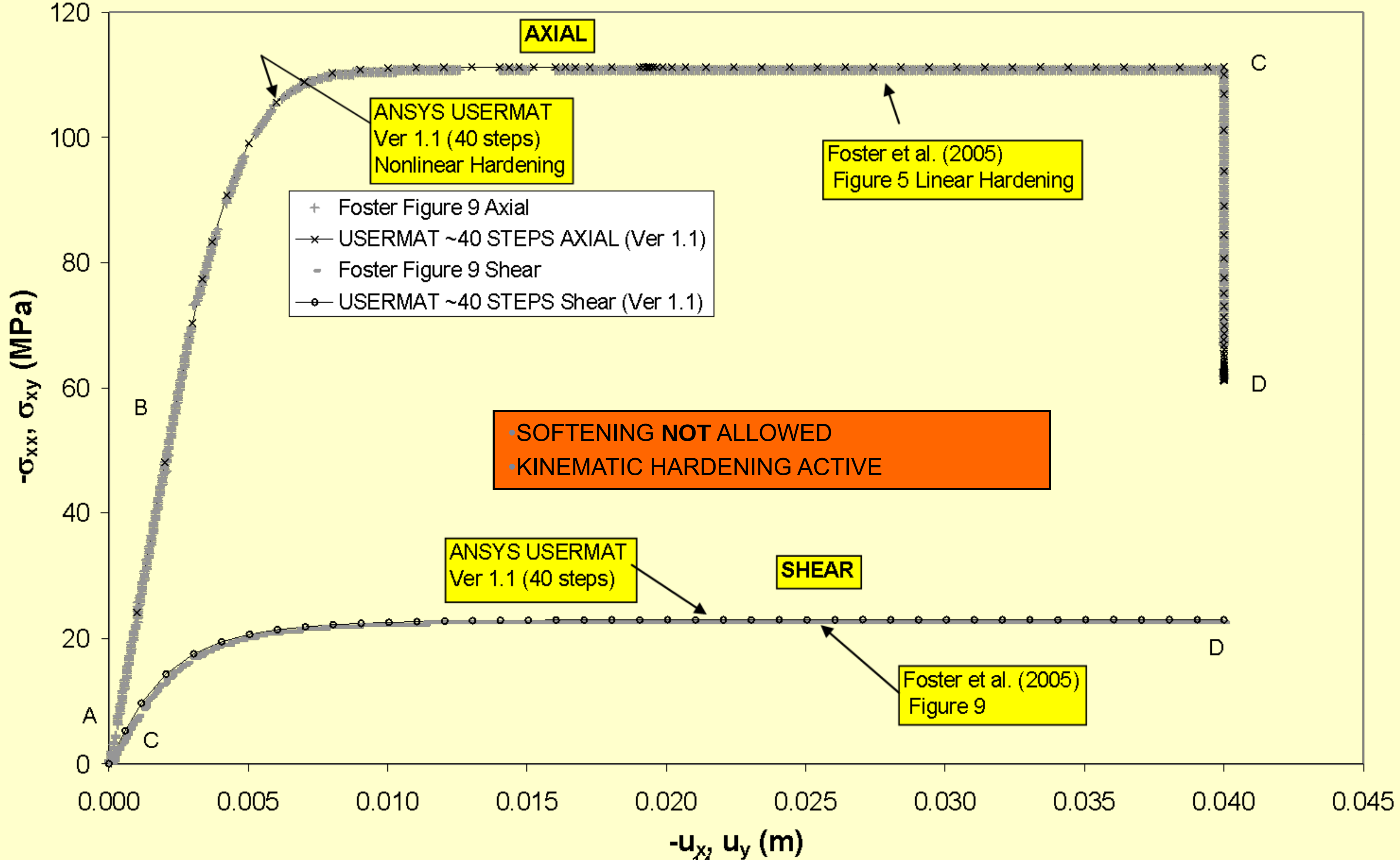
- NEGLIBLE KINEMATIC HARDENING
- NO SOFTENING ALLOWED IN USERMAT3D

Single Element Results (Example #2)

- Plane stress compression then shear.



Comparison with Foster, et al. (2005) (Example #2)



ANSYS USERMAT
Ver 1.1 (40 steps)
Nonlinear Hardening

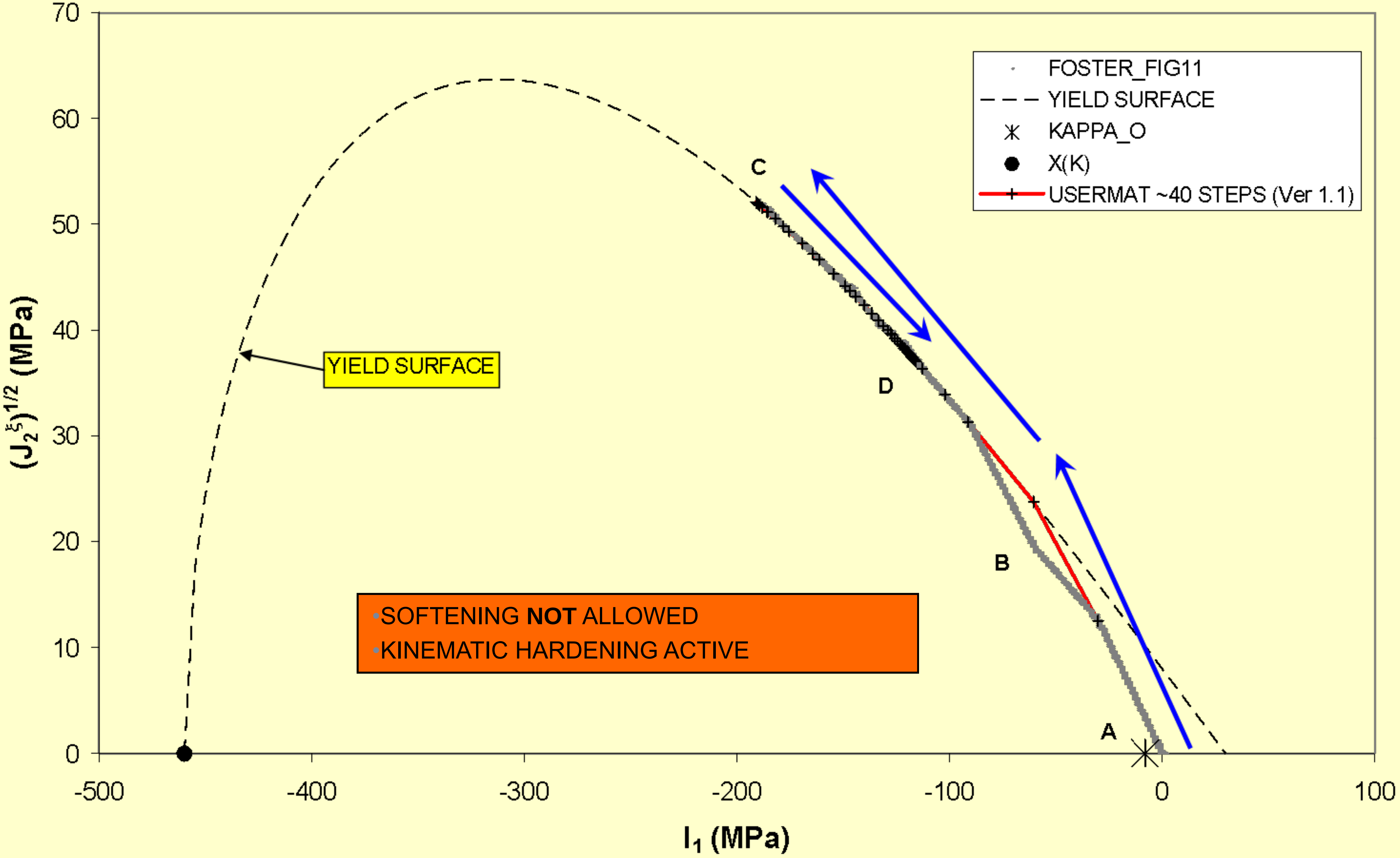
Foster et al. (2005)
Figure 5 Linear Hardening

- SOFTENING **NOT** ALLOWED
- KINEMATIC HARDENING ACTIVE

ANSYS USERMAT
Ver 1.1 (40 steps)

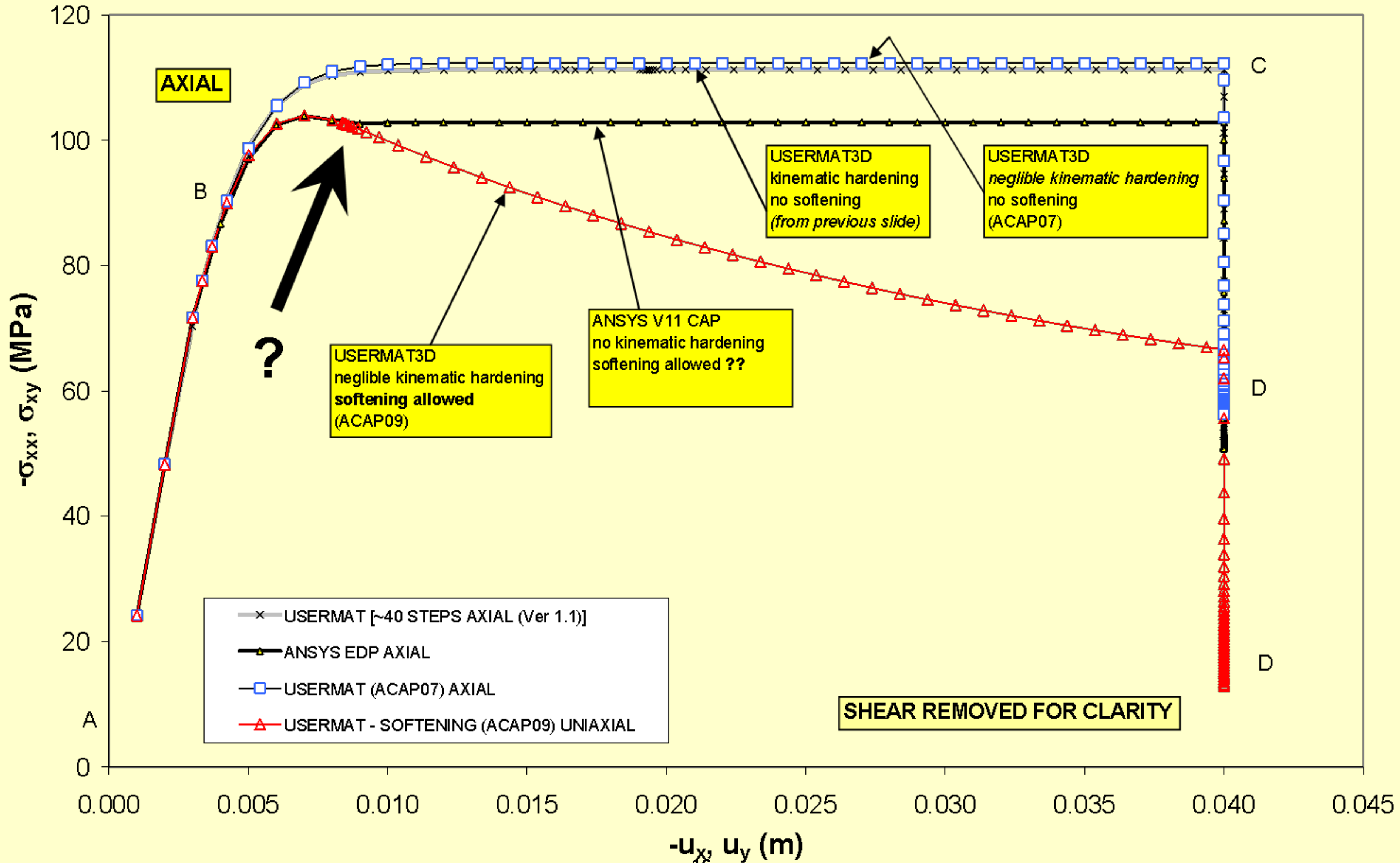
Foster et al. (2005)
Figure 9

Comparison with Foster, et al.(2005) (Example #2)

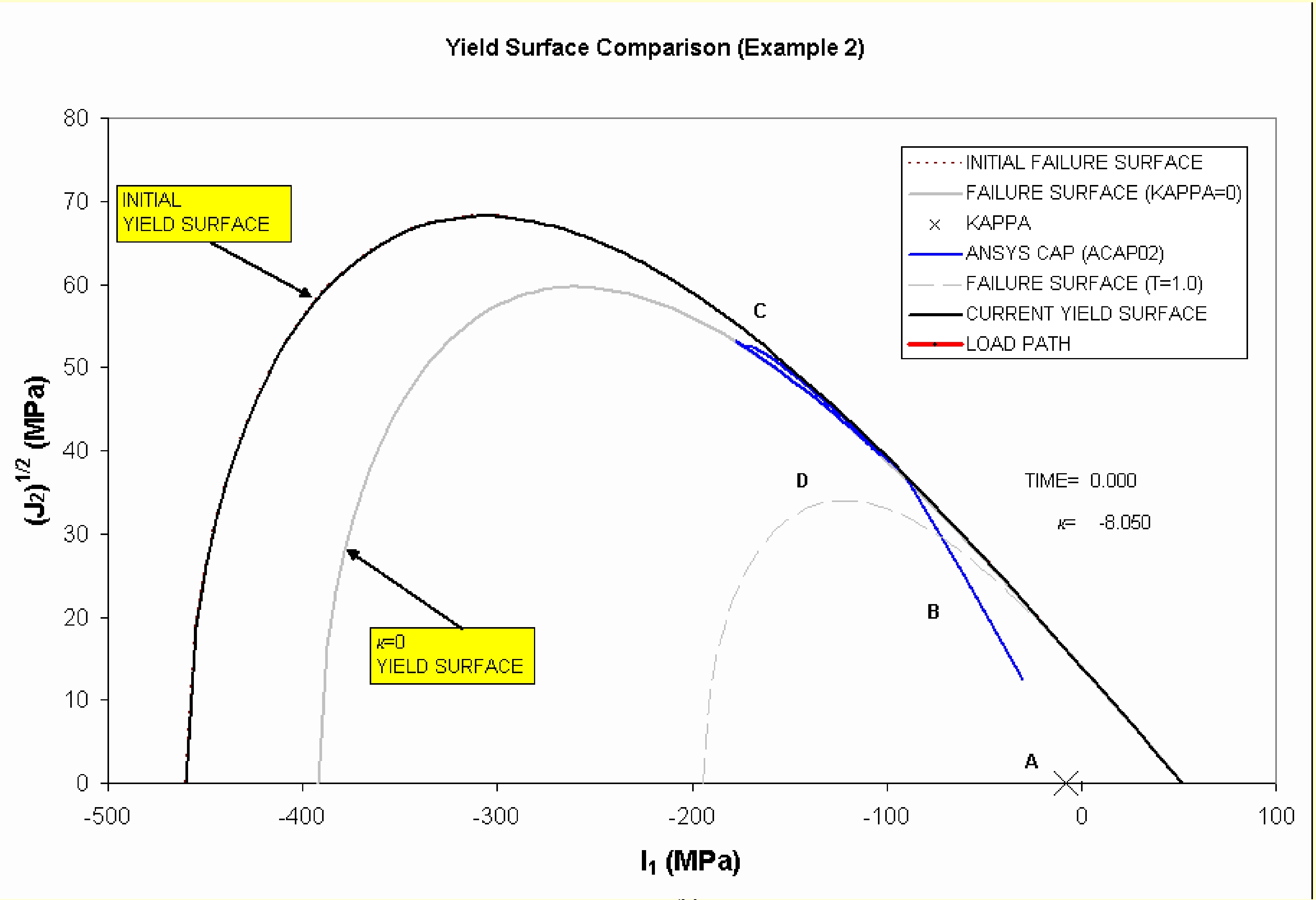


- SOFTENING **NOT** ALLOWED
- KINEMATIC HARDENING ACTIVE

Comparison with ANSYS V11 CAP (Example #2)

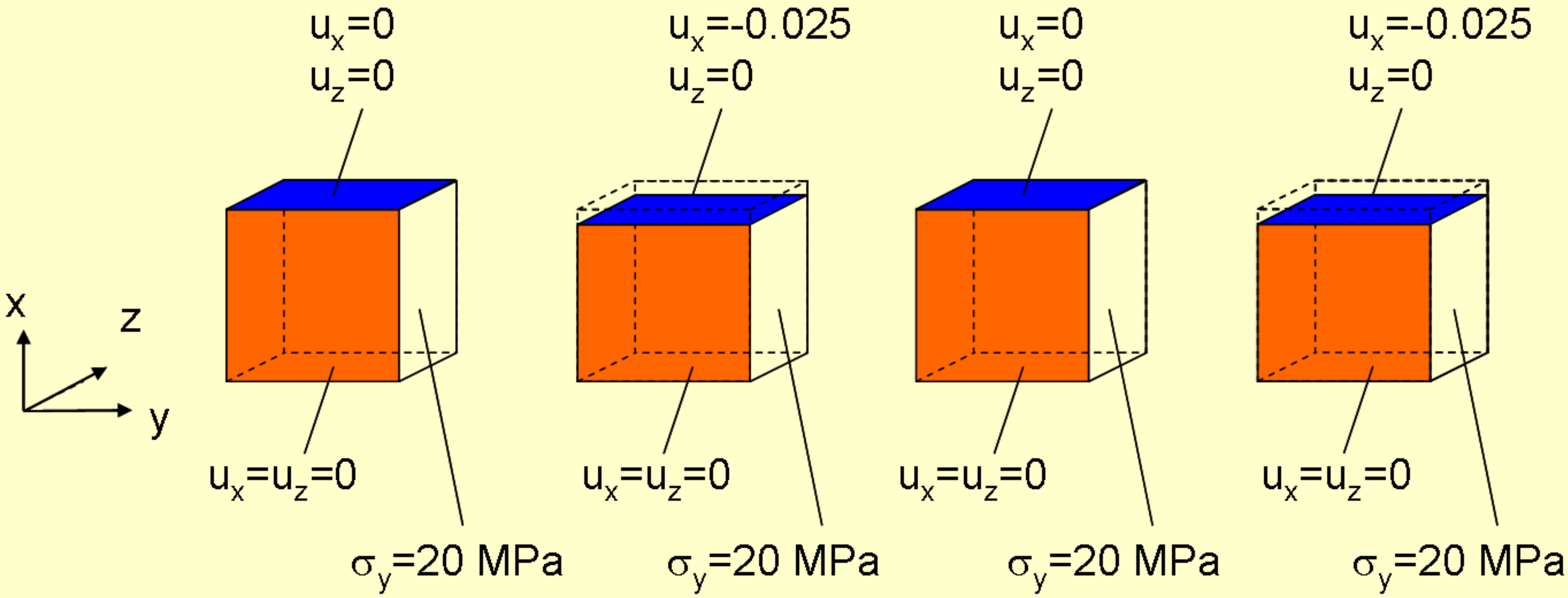


Comparison with ANSYS V11 CAP (Example #2)

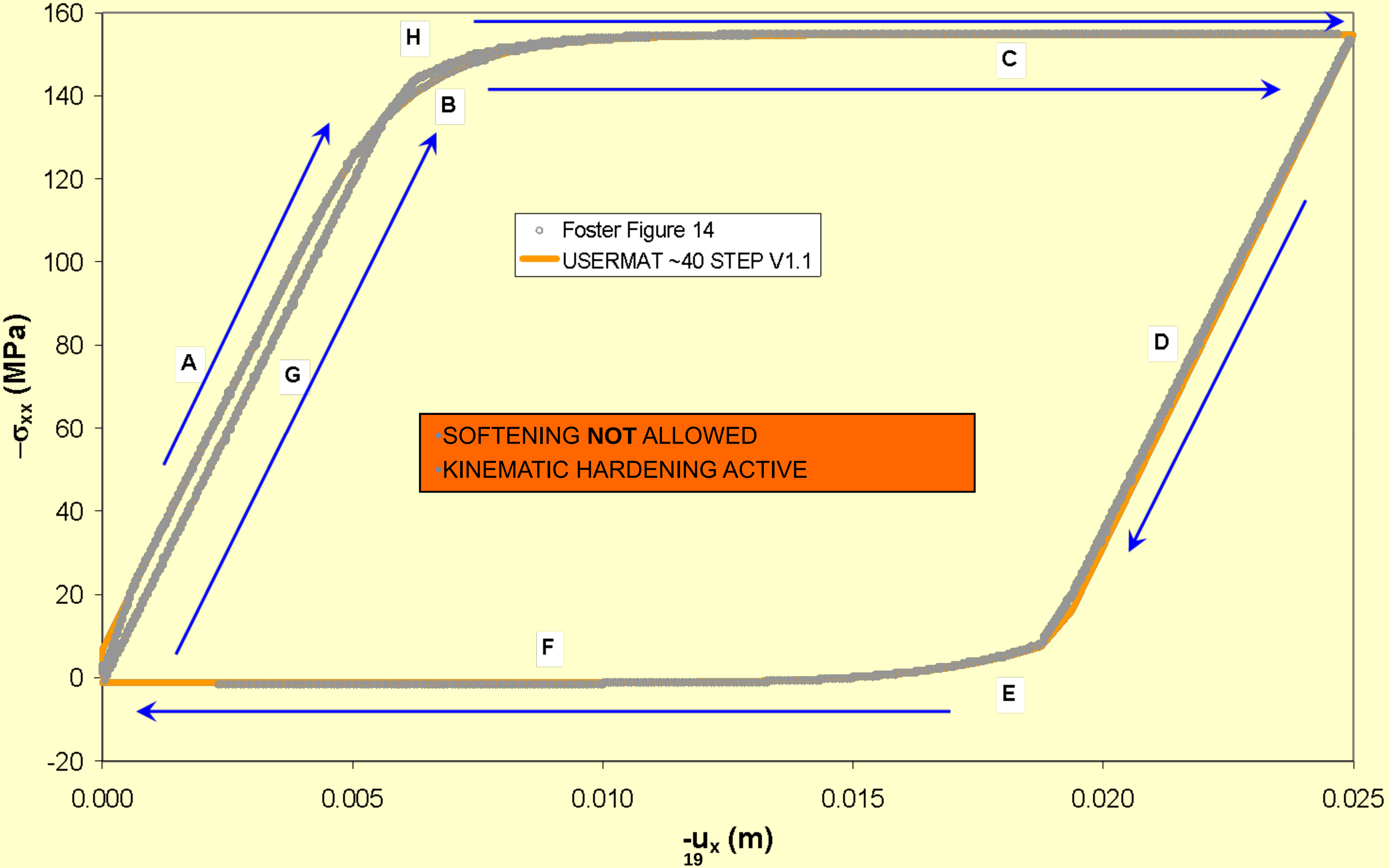


Single Element Results (Example #3)

- Plane strain with confining pressure with cycling



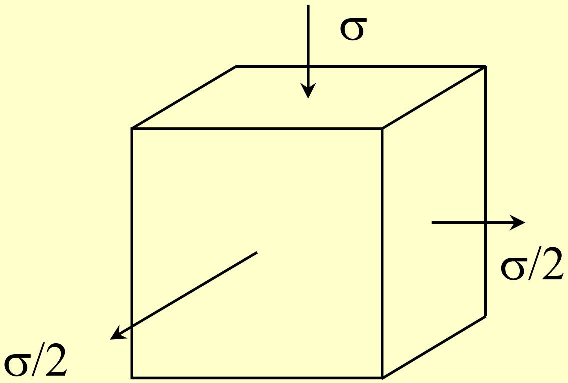
Comparison with Foster, et al. (2005) (Example #3)



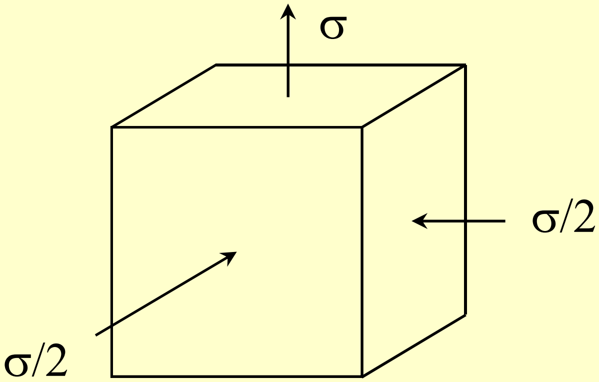
- SOFTENING **NOT** ALLOWED
- KINEMATIC HARDENING ACTIVE

Single Element Results (Example #4)

- Triaxial compression vs triaxial extension
- Constants from Box 3 of Foster, et al. (2005)

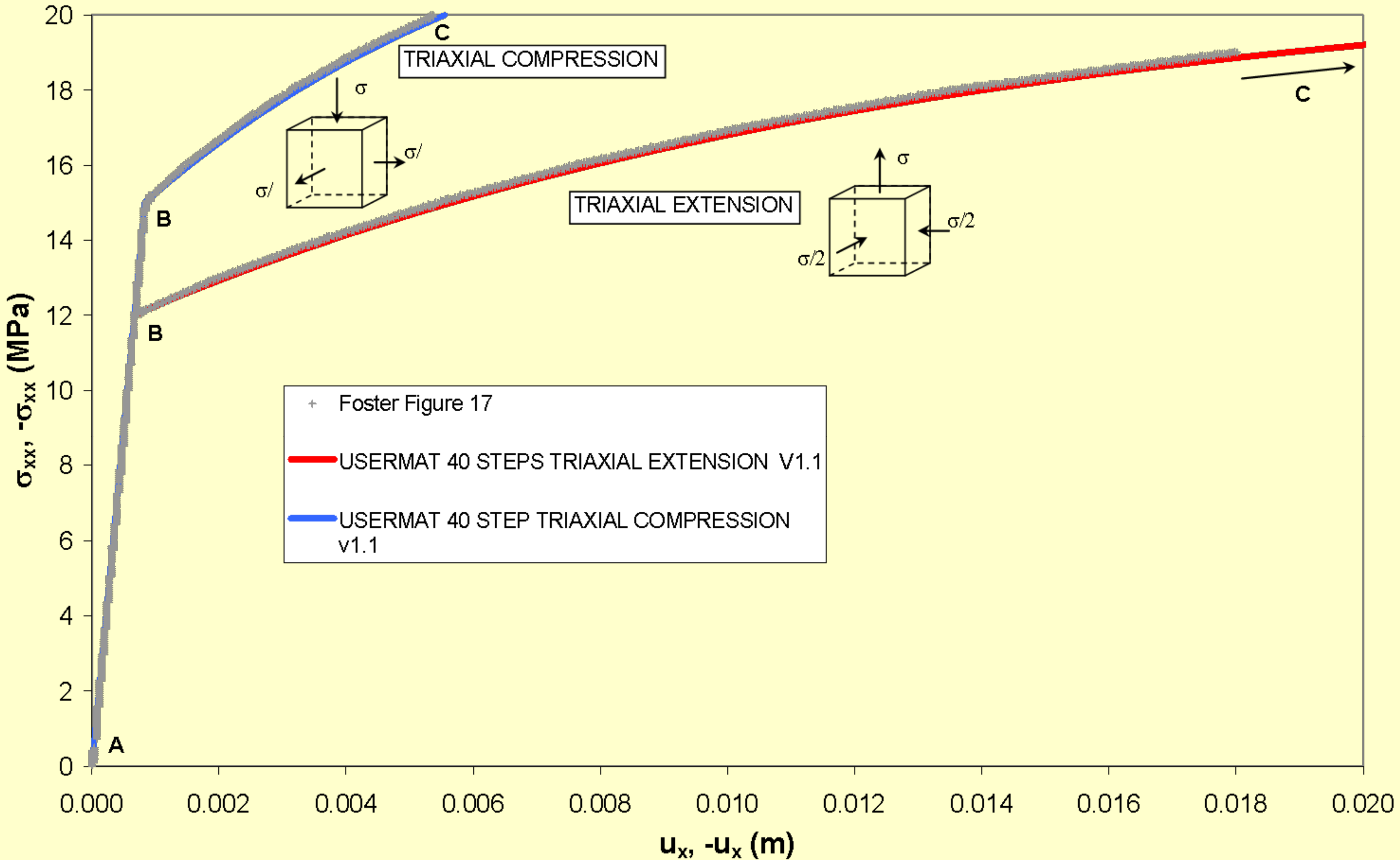


Triaxial Compression

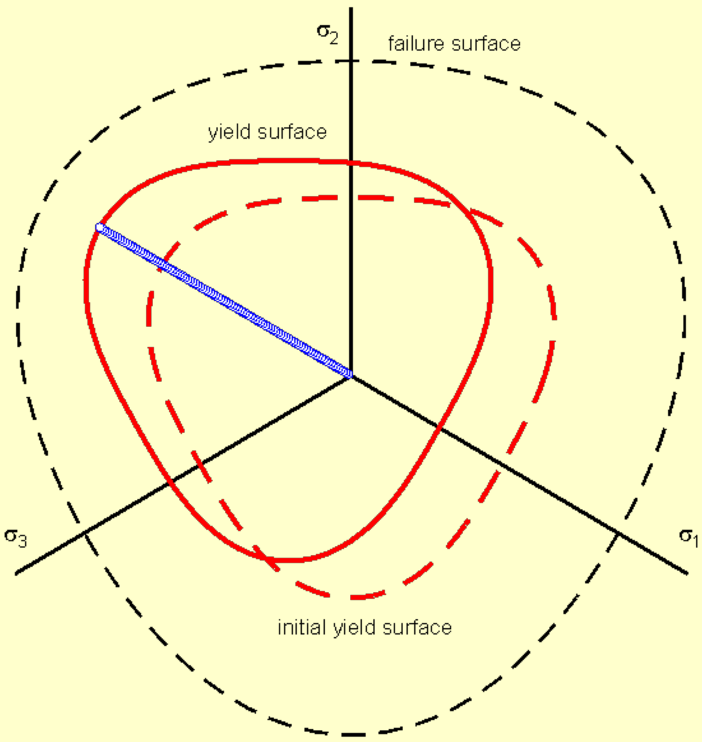


Triaxial Extension

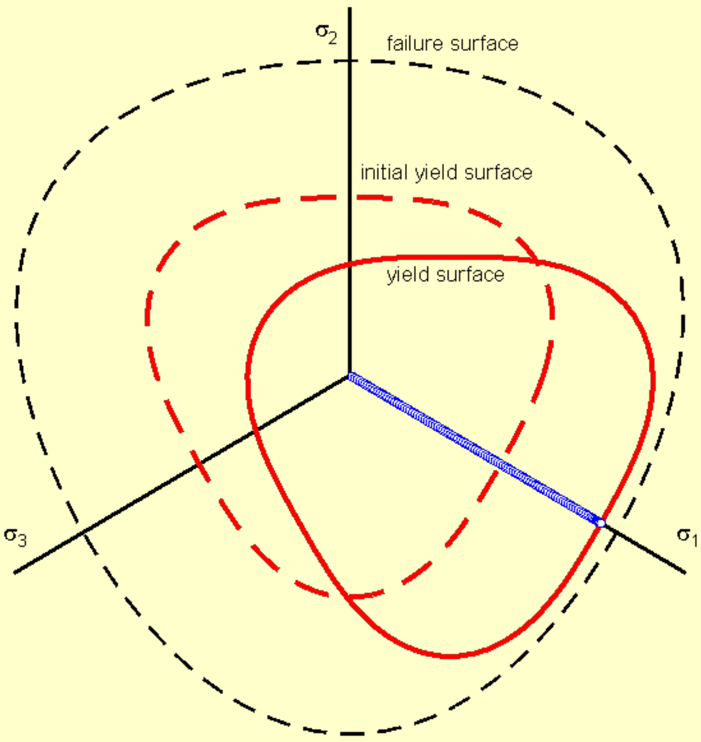
Comparison with Foster, et al. (2005) (Example #4)



π Plane Projection (Example #4)



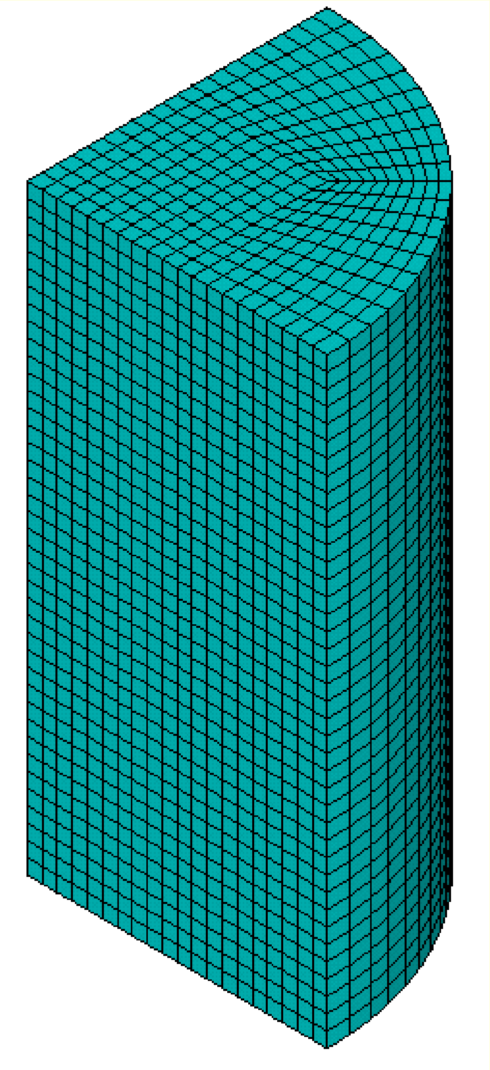
Triaxial Compression



Triaxial Extension

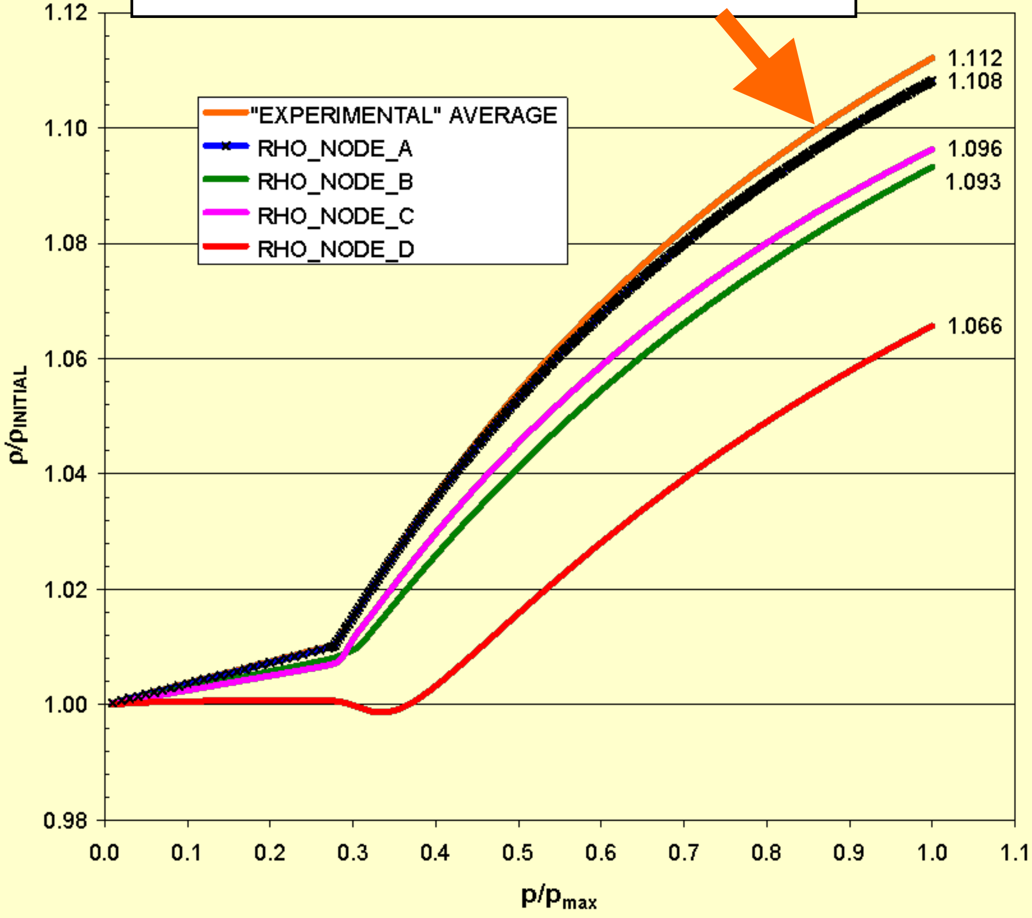
Multiple Element Results (Example #5)

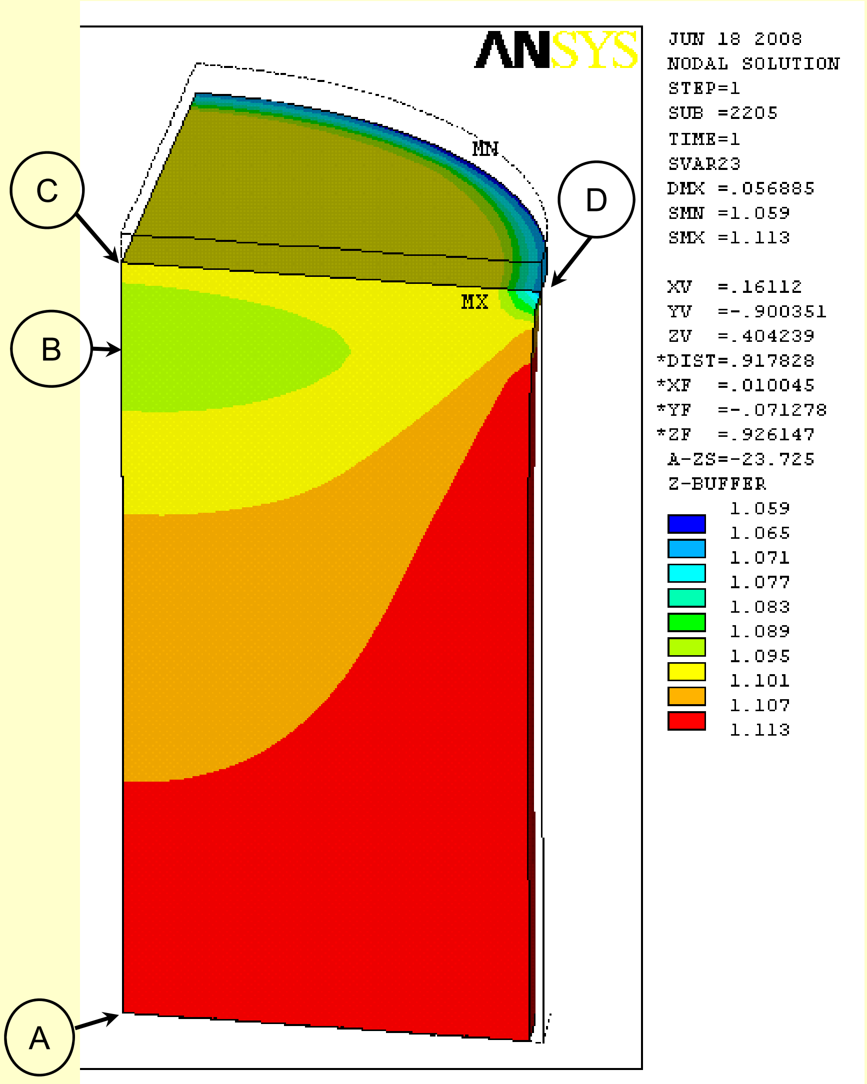
- Hydrostatic compaction
  - Cylindrical geometry
    - L/D=2 (L=3” ; D=1.5”)
    - 3 symmetry planes
  - Increasing pressure applied to sides and ends;
    80 ksi (551 MPa)
  - End nodes couple to approximate “rigid” platen with no slip conditions (high friction)
  - Salem limestone



Multiple Elements Results (Example #5’)

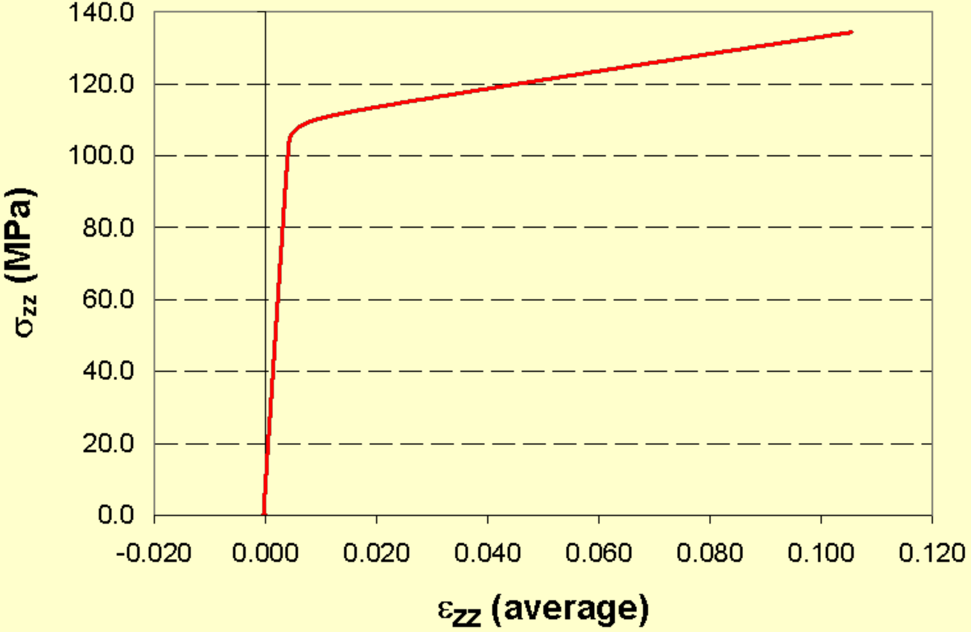
“Experimental” Average Density calculated from sample length and diameter

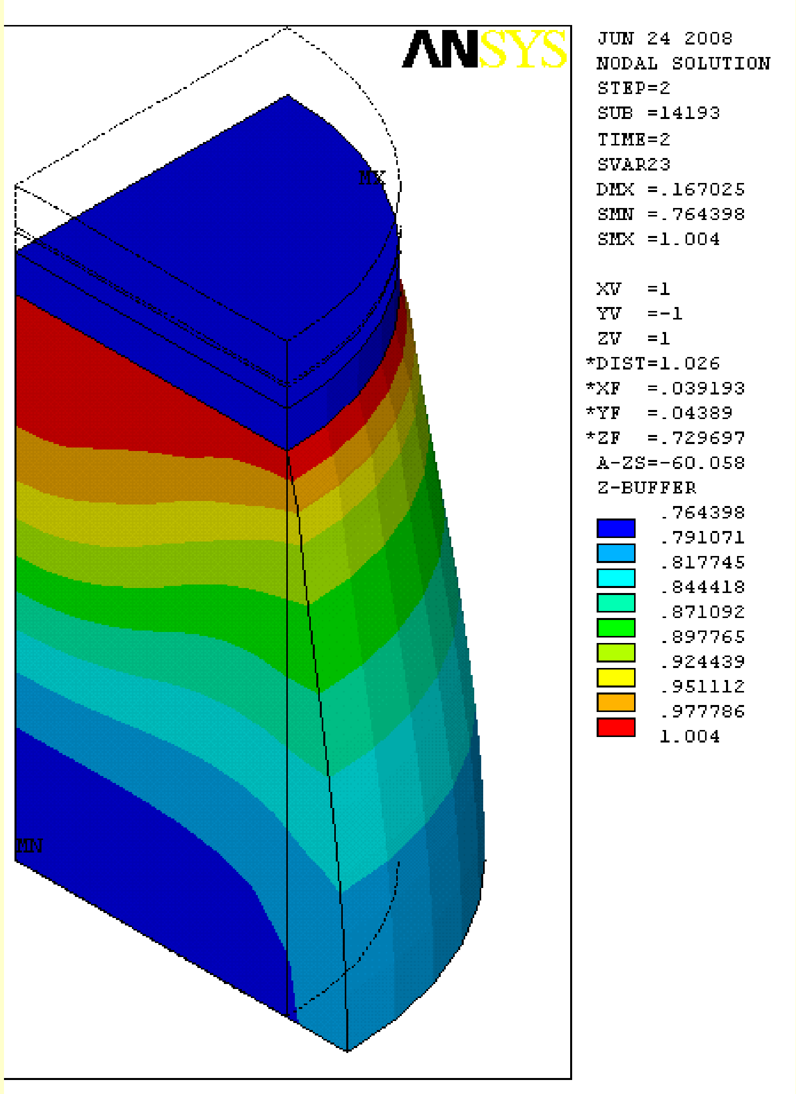




Multiple Element Results (Example #6')

- Triaxial Compaction Test
  - Salem Limestone
  - Hydrostatic confining pressure held
    (20 MPa = 2900 psi)
  - Increasing axial displacement -10%





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SUB =14193
TIME=2
SVAR23
DMX =.167025
SMN =.764398
SMX =1.004

XV  =1
YV  =-1
ZV  =1
*DIST=1.026
*XF  =.039193
*YF  =.04389
*ZF  =.729697
A-ZS=-60.058
Z-BUFFER
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      .791071
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      .951112
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      1.004
```

Summary

- An implicit formulation of a three-invariant, isotropic / kinematic hardening Cap plasticity model has been was programmed into ANSYS **V10.**
    - Agrees well with published results and ANSYS **V11** CAP results for certain conditions
    - Can predict density distributions in triaxial samples, etc.
- Usermat3d.f requires modification for computational costs
    - To handle isotropic hardening while avoiding cost of kinematic hardening
    - To handle 2D plane strain elements (**DONE**)
    - To handle changing elastic properties within each time step
- ANSYS **V11** EDP Cap model has limitations
    - No switch to prevent cap softening
    - Current tensile cap artificially limits cap softening by requiring κ to be non-positive

References

- Drucker, D.C. and Prager, W. 1952, "Soil Mechanics and Plastic Analysis or Limit Design," *Quarterly of Applied Mechanics*, Vol. 10, pp. 157-165
- Fossum, A.F. and Brannon, R.M. 2004, "Unified Compaction/Dilation, Strain-Rate Sensitive, Constitutive Model for Rock Mechanics Structural Analysis Applications" presented at Gulf Rocks 2004, the 6th North America Rock Mechanics Symposium (ARMA/NARMS 04-546).
- Foster, C.D., Regueiro, R.A., Fossum, A.F., Borja, R.I., 2005, "Implicit Numerical Integration of a Three-Invariant, Isotropic/Kinematic Hardenign Cap Plasticity Model for Geomaterials." *Computer Methods in Applied Mechanics and Engineering*, Vol 194, 5109-5138.
- Sandler, I.S. and Rubin, D. 1979, "An Algorithm and a Modular Subroutine for the Cap Model," *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol 3, 173-186
- Sandler, I.S. 2002, "Review of the Development of Cap Models for Geomaterials," 15th ASCE Engineering Mechanics Conference, June 2-5, 2002, Columbia University, New York, NY.
- Schwer, L.E. and Murray, Y.C. 1994, "A Three-Invariant Smooth Cap Model with Mixed Hardening" *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol 18, 657-688.

QUESTIONS?