

Design Optimizations Using ANSYS Generated Response Surfaces

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Abstract

This paper describes how to perform design optimizations with multiple objective functions and multiple finite element analyses using the response surfaces generated by ANSYS Probabilistic Design System (PDS). ANSYS models are parametrically built to generate response surfaces for the determinations of objective functions and state variables as functions of design variables. A numerical optimization process is employed to find the optimum solutions within the design space according to the objective functions and state variables. The optimization method presented in this paper can be used to find a common optimum design of a component used for several different applications. In general, it can be used for general design optimizations with multiple objective functions and state variables.

Introduction

Generating the response surface is a very robust built-in functionality of ANSYS PDS. The intention of this paper is to show that the response surfaces generated by ANSYS PDS can also be used for design optimizations. The major motivation of developing this optimization method is to find an optimum design of a component that is used for several different applications (or different products) for cost reduction and easier manufacturing management. Usually, different applications need separated FE (Finite Element) models since the surrounding parts contacting the common component are different among applications. The optimization method can also be used for the application with multiple objective functions; therefore, this method may be more versatile than combining several objective functions into one single objective function. This optimization method is easy to implementation as a numerical and graphical procedure is employed.

FE models have to be built parametrically with APDL to generate response surfaces for the determinations of objective functions and state variables as functions of design variables. A numerical procedure of straightforward direct search is used to find both the best solution and a region of optimum solutions for each objective function. The regions of optimum solutions are graphically represented on 2-D plots for identifying a common optimum solution for all objective functions involved.

Mathematic Descriptions of Design Optimizations

In many industrial situations, optimizing a design with multiple objective functions is needed. Such situations include designing a common component for several different applications, and optimizing the design for the application with more than one objective functions. These situations can be described in general mathematical forms as follows. The design variables are denoted by:

$$\vec{X} = [x_1, x_2, x_3, \dots, x_n] \in D^n \quad (1)$$

Where n is the total number of design variables. D^n is the n -dimensional design space, and defined by:

$$x_{i, \min} \leq x_i \leq x_{i, \max}, \quad i = 1, 2, 3, \dots, n \quad (2)$$

In above equation, $x_{i, \min}$ and $x_{i, \max}$ are the lower and upper bounds of design variables. State variables are also defined to constrain the design. The design is considered as feasible only if:

$$g_{j\alpha, \min} \leq g_{j\alpha}(\vec{X} \in D^n) \leq g_{j\alpha, \max}, \quad j = 1, 2, 3, \dots, m_\alpha \quad \& \quad \alpha = 1, 2, 3, \dots, A \quad (3)$$

Subscript α denotes the α^{th} application. A in Equation (3) is the total number of applications. m_α is the total number of state variables for the α^{th} application. $g_{j\alpha,\min}$ and $g_{j\alpha,\max}$ are the lower and upper bounds of state variables respectively. The objective functions for the α^{th} application are expressed as:

$$f_{k\alpha}(\vec{X} \in D^n), \quad k = 1, 2, 3, \dots, L_\alpha \quad \& \quad \alpha = 1, 2, 3, \dots, A \quad (4)$$

Where L_α is the total number of objective functions for the α^{th} application. The best feasible solution for one single objective function can be found by:

$$\text{Min}[f_{k\alpha}(\vec{X} \in D^n)], \quad \text{when} \quad g_{j\alpha,\min} \leq g_{j\alpha}(\vec{X} \in D^n) \leq g_{j\alpha,\max}, \quad j = 1, 2, 3, \dots, m_\alpha \quad (5)$$

A region of optimum solutions $R_{\beta\alpha}$ can be determined for the objective function by:

$$\text{Min}[f_{k\alpha}(\vec{X} \in D^n)] \leq f_{k\alpha}(\vec{X} \in D^n) \leq \text{Min}[f_{k\alpha}(\vec{X} \in D^n)] + \Delta f_{k\alpha} \quad (6)$$

$\Delta f_{k\alpha} > 0$, is a user-defined parameter used to adjust the size of $R_{k\alpha}$. The region of optimum solutions $R_{k\alpha}$ is also n -dimensional, and can be expressed by:

$$R_{k\alpha}(\vec{X} \in D^n), \quad k = 1, 2, 3, \dots, L_\alpha \quad \& \quad \alpha = 1, 2, 3, \dots, A \quad (7)$$

The common optimum solution for all objective function is a compromise of $R_{k\alpha}$ ($k=1, 2, 3, \dots, L_\alpha$ & $\alpha=1, 2, 3, \dots, A$).

Determinations of Objective Functions and State Variables

Both objective functions and state variables are dependent variables. The objective functions and state variables as functions of design variables are explicitly determined by the response surface functionality of ANSYS PDS. Parametrically built FE models are required. It is extremely important to make sure that the response surfaces sufficiently fit the underlying FE results. Extrapolations of the response surface outside the design space used to determine the response surface should be avoided.

Numerical Procedure of Design Optimizations

A numerical procedure of straightforward direct search is employed to optimize multiple objective functions. The whole design space is first represented by discrete points that are evenly spaced in each dimension. The objective functions and state variables are then mapped to the points. The values of objective functions and state variables are examined at all points per Equations (3), (5), and (6) to determine $R_{k\alpha}$.

The common optimum solution may be determined mathematically. If $[x_1, x_2, x_3, \dots, x_n]_{k\alpha}$ is the best solution within $R_{k\alpha}$ ($k=1, 2, 3, \dots, L_\alpha$ & $\alpha=1, 2, 3, \dots, A$), and $d_{k\alpha}$ is the distance from the common optimum solution to $[x_1, x_2, x_3, \dots, x_n]_{k\alpha}$, therefore, the common optimum solution might be found by:

$$\text{Min} \left[\sum_{\alpha=1}^A \sum_{k=1}^{L_\alpha} \omega_{k\alpha} d_{k\alpha}^2 \right] \quad (8)$$

Where $\omega_{k\alpha}$ is weighting function. Since engineering judgments are usually required to identify a common optimum solution, a graphic method is thus used and described as follows. All regions of optimum solutions, $R_{k\alpha}$ ($k=1, 2, 3, \dots, L_\alpha$ & $\alpha=1, 2, 3, \dots, A$), are plotted together on a matrix of 2-D plots with all possible combinations of two design variables out of n design variables. The common optimum solution can be visually identified from the 2-D plots. To assist in finding the common optimum solution, $\Delta f_{k\alpha}$ can be properly adjusted to change the sizes of $R_{k\alpha}$.

Note that it is not always possible to find a feasible common optimum solution. Even though a feasible common optimum solution is possible, it is not necessary the solution is practical. Several successive iterations of the optimization process with modified design space and FE models may be necessary to reach

a final desirable solution. Furthermore, the optimum solution does not necessarily satisfy the reliability requirement. The evaluation of design reliability could be a different issue that is not covered in this paper.

An Example to Demonstrate the Optimization Method

To demonstrate the use of this optimization method, an example is given below. In this example, the design space has three design variables. One mechanical component is used for two different applications (or two products), and each application has one objective function and two state variables. This example optimization problem can be mathematically expressed by:

Design space :

$$\vec{X} = [x_1, x_2, x_3] \quad (9)$$

$$\left. \begin{array}{l} x_{1,\min} \leq x_1 \leq x_{1,\max} \\ x_{2,\min} \leq x_2 \leq x_{2,\max} \\ x_{3,\min} \leq x_3 \leq x_{3,\max} \end{array} \right\} D^3$$

Application #1 :

$$\begin{aligned} \text{Min} [f_{11}(\vec{X} \in D^3)] \leq f_{11}(\vec{X} \in D^3) \leq \text{Min} [f_{11}(\vec{X} \in D^3)] + \Delta f_{11} &\Rightarrow R_{11} \\ g_{11,\min} \leq g_{11}(\vec{X} \in D^3) \leq g_{11,\max} & \\ g_{21,\min} \leq g_{21}(\vec{X} \in D^3) \leq g_{21,\max} & \end{aligned} \quad (10)$$

Application #2 :

$$\begin{aligned} \text{Min} [f_{12}(\vec{X} \in D^3)] \leq f_{12}(\vec{X} \in D^3) \leq \text{Min} [f_{12}(\vec{X} \in D^3)] + \Delta f_{12} &\Rightarrow R_{12} \\ g_{12,\min} \leq g_{12}(\vec{X} \in D^3) \leq g_{12,\max} & \\ g_{22,\min} \leq g_{22}(\vec{X} \in D^3) \leq g_{22,\max} & \end{aligned} \quad (11)$$

To determine the objective functions and state variables, separated FE models are built parametrically for Applications #1 and #2. The ANSYS commands for declaring input variables and output parameters in ANSYS PDS are listed in Figure 1. The input variables are design variables, and output parameters are objective functions and state variables. The distribution types for input variables are irrelevant to the optimization processes, and any types of distribution can be used. The response surface method with design of experiment method is adopted. The response surface approximates the output parameter as a function of input variables. Therefore the response surfaces can be used for the determinations of objection functions and state variables. The general mathematical expression for the response surfaces used in this example is given in Figure 2. After the completion of PDS runs, the response surfaces are determined by performing regression analyses. Refer to ANSYS documentation for guidelines of how to fit a response surface and how to evaluate the quality (goodness-of-fit) of response surface. All necessary regression results regarding the response surface are saved to a text file for numerical optimizations. The response surface for the objective function of application $\alpha=1$ is plotted in Figure 3.

For numerical optimization, the design space is represented by discrete points that are evenly spaced in all dimensions. The values of objection functions and state variables are mapped to the points and examined for optimum solutions, as illustrated in Figure 4. The optimization results are plotted in a matrix of 2-D plots in Figure 5 for the determination of the common optimum solution. Increase of total number of points within the design space can enhance resolution (or accuracy). For the same purpose, multiple optimization executions with increasingly reduced design space that includes R_{ka} can also increase accuracy.

```

! ----- Declare random input variables -----
PDVAR, X1, UNIF, X1_min, X1_max
PDVAR, X2, UNIF, X2_min, X2_max ← Design Variables
PDVAR, X3, UNIF, X3_min, X3_max

! ----- Declare output parameters -----
PDVAR, f11, RESP ← Objective function
PDVAR, g11, RESP ← State variables
PDVAR, g21, RESP

```

Figure 1. ANSYS commands for declaring input variables and output parameters for application $\alpha=1$. The input variables are design variables. The output parameters are objective functions and state variables.

```

X1_scaled = C1 * X1 + C2
X2_scaled = C3 * X2 + C4
X3_scaled = C5 * X3 + C6

F11_trans = C7
F11_trans = F11_trans + C8 * X1_scaled
F11_trans = F11_trans + C9 * X2_scaled
F11_trans = F11_trans + C10 * X3_scaled
F11_trans = F11_trans + C11 * X1_scaled * X1_scaled
F11_trans = F11_trans + C12 * X2_scaled * X2_scaled
F11_trans = F11_trans + C13 * X3_scaled * X3_scaled

F11_trans = F11_trans + C14 * X1_scaled * X2_scaled
F11_trans = F11_trans + C15 * X1_scaled * X3_scaled
F11_trans = F11_trans + C16 * X2_scaled * X3_scaled

F11 = 10**F11_trans

```

Figure 2. General mathematical expression of response surface (objective function of application $\alpha=1$). C1 to C16 are regression coefficients determined by ANSYS PDS. Refer to ANSYS documentation for detailed technical information.

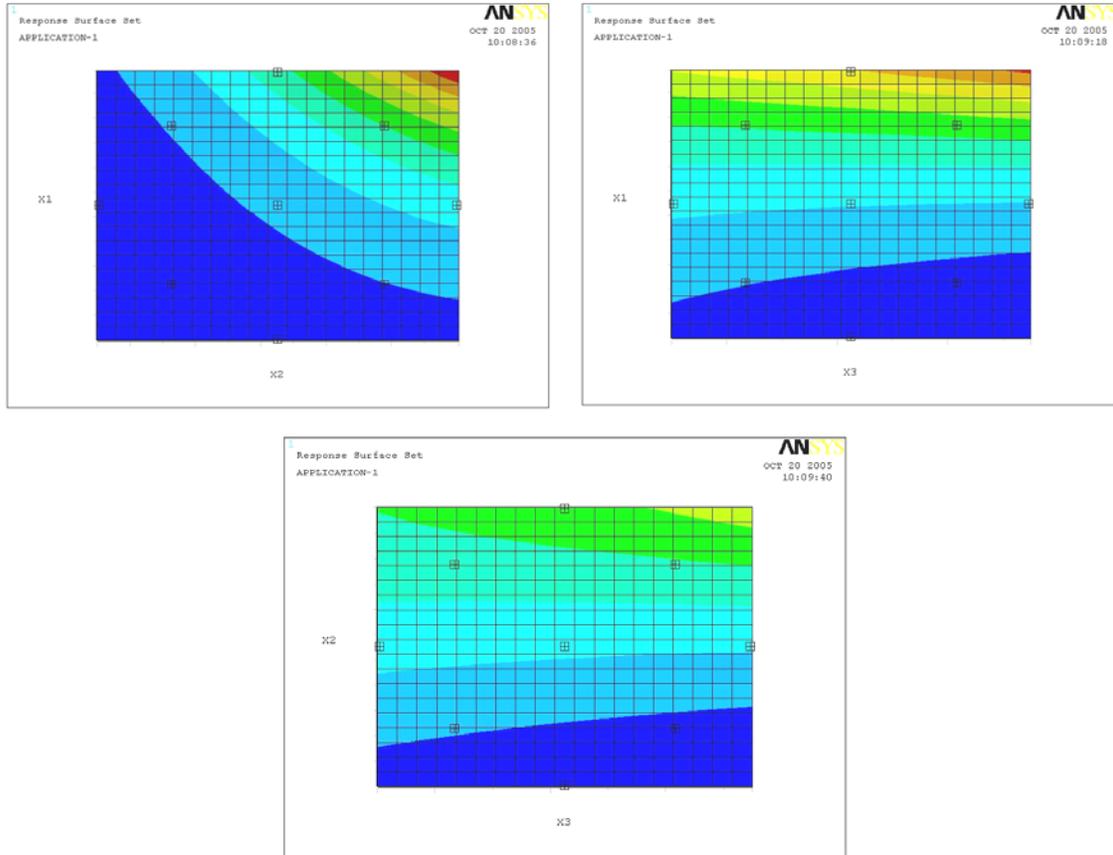


Figure 3. Response surface plots for the objective function of application $\alpha=1$

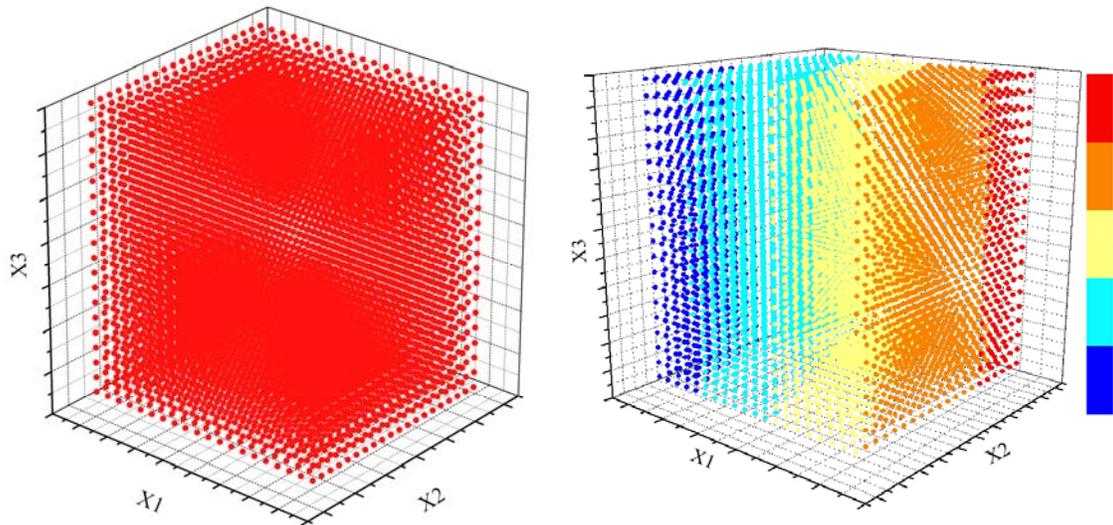
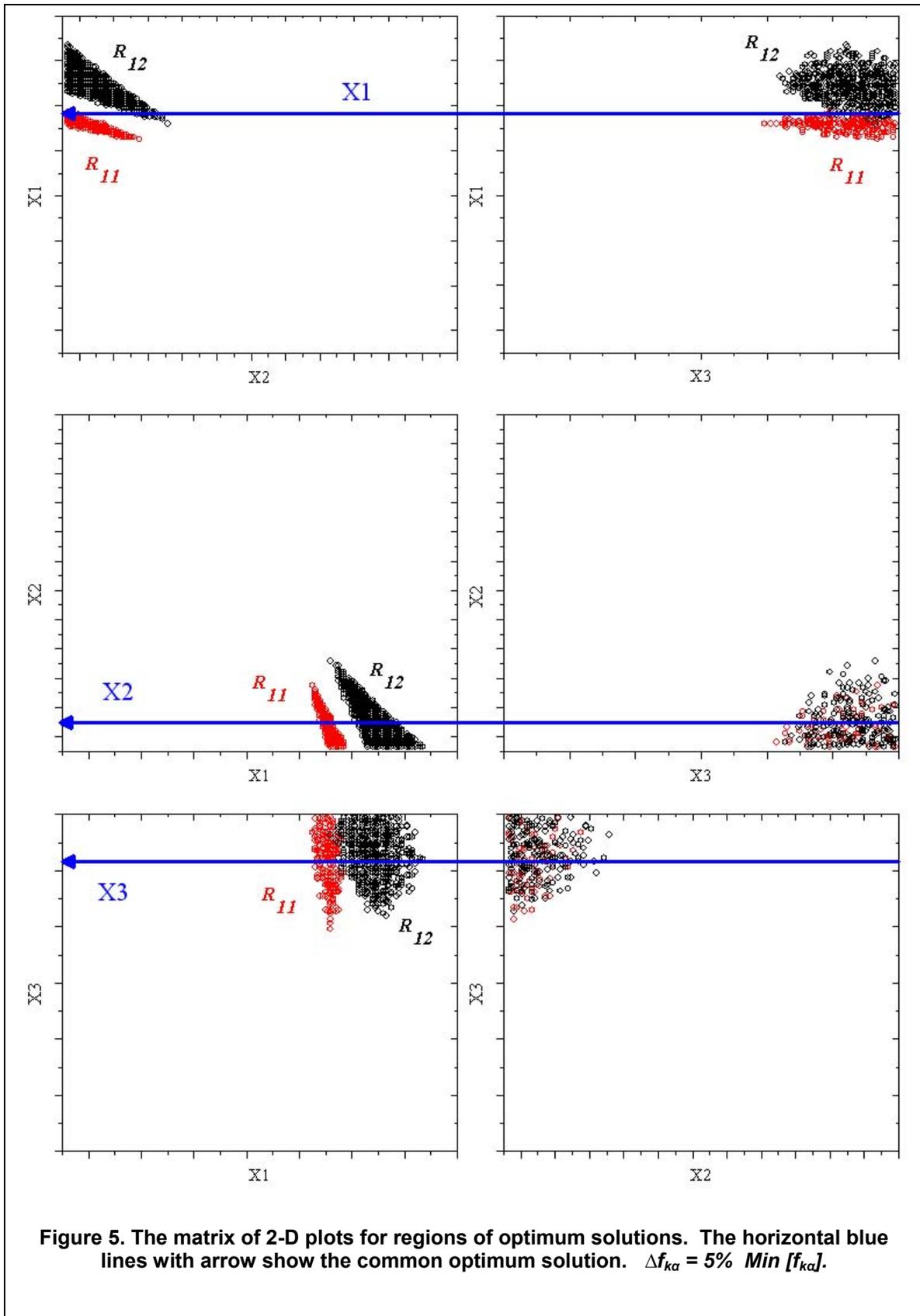


Figure 4. Design space is represented numerically by points evenly spaced in all design variables as shown in the left figure above. The objective functions and state variables are mapped to the points, see an example in the right figure above. The objective functions and state variables are examined at each point for optimum solutions.



Summary

An optimization method is developed based on the objective functions and state variables generated by ANSYS PDS response surface functionality as functions of design variables. Parametrically built FE models are required for generating response surfaces. A numerical and graphical procedure is described to optimize design according to objective functions and state variables within design space. The numerical procedure can be programmed either outside ANSYS or inside ANSYS using ANSYS APDL language. An example is presented in this paper to illustrate the use of this optimization method.

The optimization procedure can yield both the best solution and a region of optimum solutions for each objective function. Since simplifications and assumptions are usually involved in analyses, compared to the single best solution, a region of optimum solutions could be more meaningful and more useful to find a practically more feasible solution. The best solution obtained from this optimization method for a single objective function is very close to the best solution using ANSYS Design Optimization, verifying the method.

Furthermore, the optimization method presented in paper can solve optimization problems with multiple objective functions and multiple FE models. Therefore, the method can be used to find a common optimum design of a component used for several different applications.

In the future works, the points representing the design space can be positioned according to the gradients of objective functions and state variables to improve the efficiency and accuracy of the numerical optimization process. Besides graphic method to identify the common optimum solution, a more elegant algorithm could be developed to determine the common optimum solution, which could take into account the location of the best solution, the gradient, and boundary of the region of optimum solutions for each of all objective functions involved.

References

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