Evaluation of the Non-Linear Fracture Parameters J and C* with ANSYS

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Abstract

Currently ANSYS has the capability of evaluating the linear elastic fracture parameter K by using quarter-point crack-tip elements. This capability has been extended to the evaluation of the non-linear fracture parameters J (for modeling plasticity) and C* (for modeling creep crack growth) by the development of a post-processor, and a modified crack-tip element. The theoretical background to the post-processor and the element is described, together with some two-dimensional verification examples of a fracture test specimen.

Introduction

In such structures as nuclear pressure vessels and turbine blades it is essential to be able to accurately predict the behavior of cracks, both actual and hypothetical. At present in ANSYS the linear elastic fracture mechanics (LEFM) parameter K can be computed in two and three-dimensions. However if significant plasticity occurs at the crack tip then the path-independent J-integral is a more appropriate fracture parameter. Further, for structures operating at high temperature the C*-integral is the appropriate parameter for predicting creep crack growth rates. These two line integrals are closely related and may be computed using similar methods.

A Fortran post-processor has been developed for ANSYS which reads the results from a stress analysis run and computes the appropriate line integral along a path through the integration points. The processor computes the integrals in both two and three-dimensions but this paper is restricted to the former. Under limited conditions (which will be identified) the J-integral may be evaluated with an ANSYS macro.

Initially the theoretical background of the two integrals will be summarized and then their implementation within the post-processor will be described. Their use will be illustrated with a simple fracture test specimen and results compared with other published data where possible.

Theoretical Background

Stress Intensity Factor K

In LEFM the stress intensity factor K is the relevant fracture parameter to characterize the stress and strain fields around the crack tip, as originally described by Irwin [1]. Under mode I (crack-opening) loading KI may be compared with a material's fracture toughness KIC in order to predict the stability of a crack. To compute KI with the finite element method (FE) quarter-point crack-tip elements were introduced by Barsoum [2]. These triangular elements are illustrated in figure 1 and are available in ANSYS in two dimensions by modifying the PLANE82 element using the KSCON command. The nodes at the quarter-points adjacent to the crack tip ensure that the displacements in the near-tip region are proportional to $\sqrt{r}$ (where r is the distance from the crack tip), which is in agreement with [1].
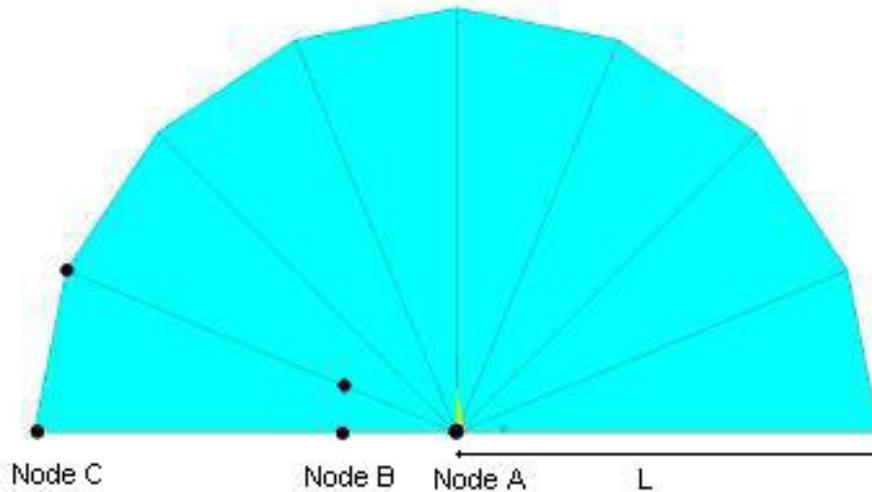
Figure 1. LEFM Crack-tip Elements

On completion of the FE analysis the mode I stress intensity factor for a cracked model with symmetry about the crack plane is derived from Aliabadi and Rooke [3] as

$$K_I = \frac{2\mu}{\kappa+1}\sqrt{\frac{2\pi}{l}}(4u_2^B - u_2^C)\ldots\ldots(1)$$

where $\mathbf{u_2}^{\mathbf{B}} =$

Displacement in x_2 direction at node B,

$\mathbf{l} =$

element dimension,

$\mu, E =$

shear modulus and Young's modulus,

$\kappa =$

$(3-4\nu)$, where ν is Poisson's ratio.

J Contour Integral

The post-yield fracture mechanics (PYFM) parameter J-integral was introduced by Rice [4], and discussed with relation to fracture toughness by Landes & Begley [5]. It was found to characterize the stress and strain fields surrounding a crack tip with significant plasticity, and also to correlate with fracture toughness JIC test data. In its simplest form, and with reference to figure 2, J around the anti-clockwise path Γ is given by

$$J = \int_{\Gamma}(Wn_1 - \sigma_{ij}\frac{\partial u_i}{\partial x_1}n_j)ds\ldots\ldots(2)$$

where the strain energy density W is given by

$$W = \int_0^{\sigma_{ij}}\sigma_{ij}d\varepsilon_{ij}\ldots\ldots(3)$$

and $\sigma_{ij}, \varepsilon_{ij}, u_i =$

stresses, strains and displacements,
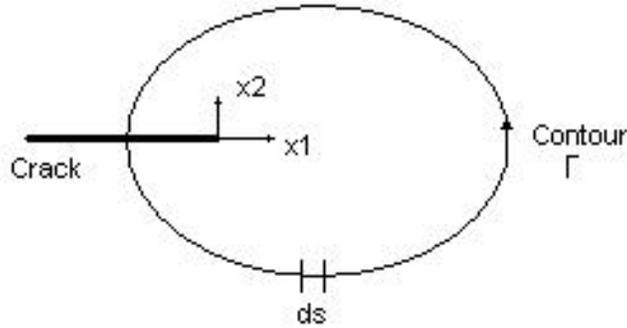
$\mathbf{n_i} =$

normal to path in x_i direction.

Figure 2. J-integral Path Definition

In this form it is possible to evaluate J under elastic conditions with an ANSYS macro [6] in which the stresses and strains are obtained using the ETABLE command. However, for a number of reasons such a macro is of limited use: firstly, greater accuracy is obtained using integration point results which are not available with the ETABLE command; and secondly, equation (2) is not applicable to axisymmetric models and, more importantly, where thermal loading occurs. For axisymmetry Bergkvist and Huong[7] extended equation (2) which becomes

$$J_{AX} = J + \iint_A \frac{1}{r}(\sigma_\phi \varepsilon_\phi - \sigma_r \varepsilon_r - \sigma_{rz} \frac{\partial u_z}{\partial r})dA \ldots\ldots(4)$$

where $\mathbf{A} =$

area within J contour path.

The area integral is difficult to evaluate within an ANSYS macro, thus the need for a post-processor. Additionally with thermal loading , Ainsworth, Neale and Price [8] extended equation (2) to

$$J_\theta = J + \iint_A \sigma_{ij} \frac{\partial \theta_{ij}}{\partial x_1} dA \ldots\ldots(5)$$

in which the strain energy density W becomes

$$W = \int \sigma_{ij} d(\varepsilon_{ij} - \theta_{ij}) \ldots\ldots(6)$$

where $\theta_{ij} =$

thermal strains.

For analysing the thermal loading of axisymmetric bodies equations (4) and (5) may be combined. In addition equations (2) to (6) are applicable to both elastic and plastic material behavior.

C Contour Integral*

The J-integral concept may also be extended into the creep regime where J is replaceable by the C(t) fracture parameter. In the long term as a body at high temperature reaches steady-state conditions C(t) becomes the parameter C*, which is not a function of time. As with J the parameters C(t) and C* have been

shown to characterize the stress and strain fields in the fracture zone at the crack tip and may be related to creep crack growth rate data. For power-law creep, see for example Boyle & Spence [9], where the creep strain rate is given by

$$\dot{\varepsilon}_c = B\sigma^n \ldots\ldots(7)$$

where **B, n** =

creep constants for specified temperature,

the parameter C(t) may be evaluated by an expression analogous to equation (2), from Li, Needleman & Shih [10]

$$C(t) = \int_{\Gamma} (\frac{n}{n+1}\sigma_{ij}\dot{\varepsilon}_{ij}n_1 - \sigma_{ij}\frac{\partial \dot{u}_i}{\partial x_1}n_j)ds \ldots\ldots(8)$$

Equation (8) may be extended for thermal loading and axisymmetric structures in the same manner as the J-integral.

The above equations form the basis of the ANSYS post-processor to be discussed. For 2D and 3D cracked structures J may be evaluated with both elastic and plastic problems, and also C(t) and C* in the creep regime. These integrals are theoretically path-independent, but under some conditions it is necessary to select the path carefully in order to evaluate the integral accurately.

PYFM Crack-tip Element

The quarter-point element of LEFM is not appropriate for analysing cracks where significant plasticity occurs on loading. This is because the displacements in the crack-tip zone are now proportional to r rather than $\sqrt{r}$, and also because crack-tip blunting occurs. Li, Shih & Needleman [11] described a suitable crack-tip element for plasticity which is shown in the figure 3. This element is based on the ANSYS PLANE82 element, appears to be triangular, but actually has eight nodes. Three of the nodes are coincident, but independent, and located at the crack tip. On loading (as shown in the figure 4) the crack-tip nodes separate and simulate crack blunting. Unlike the previously described quarter-point element all mid-side nodes are retained.
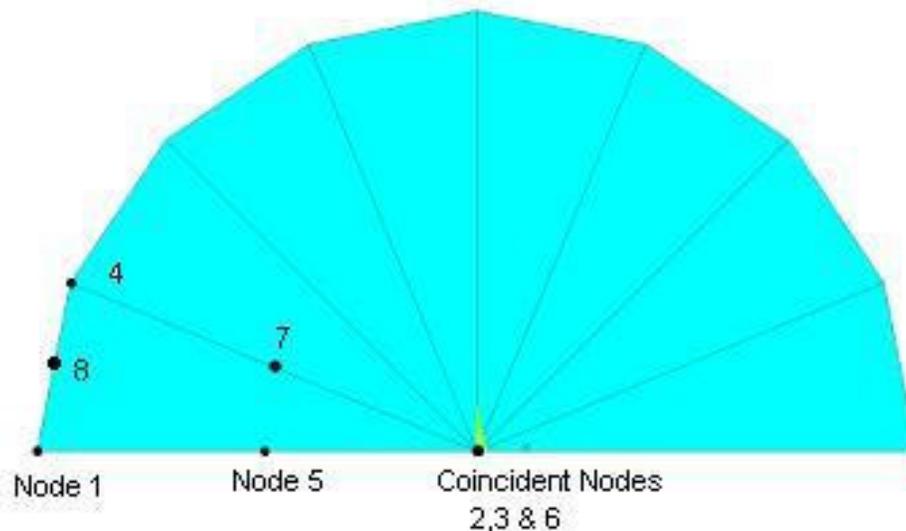

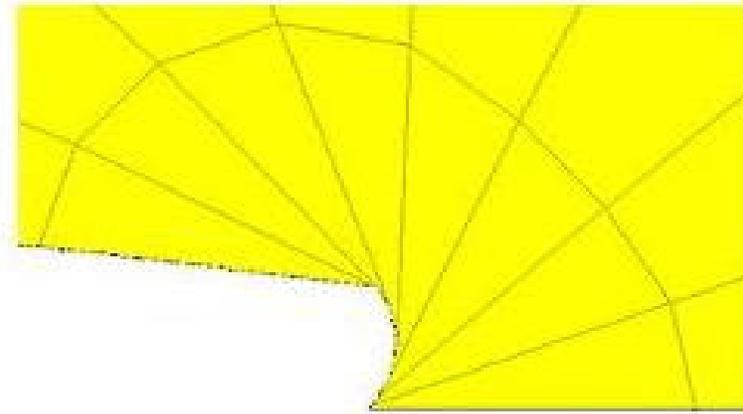**Figure 3. PYFM Crack-tip Elements**

Figure 4. Behaviour of Coincident Nodes at Crack Tip

These 'collapsed' elements have been utilized for all the plastic and creep analyses in the present study. They have been generated by switching off element shape checking in ANSYS.

Numerical Implementation

The fracture post-processor which has been developed reads the results from an elastic, plastic or creep stress analysis run. In particular the stresses and strains at the integration points of each element are required, together with the nodal displacements.

Once the J, or C(t) path has been defined the contribution from each element on the path may be computed. With reference to figure 5, the path passes through two of the four existing integration points. This allows the integration point data from ANSYS to be used directly in evaluating dJ - the contribution of the element to J - from equation (2). Using Gaussian quadrature dJ is given by

$$dJ = \sum_{m=1}^{2} \left[W(\xi,\eta)n_1 - \sigma_{ij}(\xi,\eta)\frac{\partial u_i(\xi,\eta)}{\partial x_1} n_j \right]_m w_m \ldots\ldots(9)$$

where $\xi,\eta =$

local coordinates in element ($\pm$ 0.5774 at integration points),

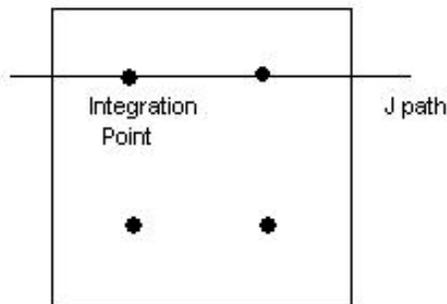$w_m =$

weight for Gaussian quadrature = 1.0.

Figure 5. J Path Through Plane82 Element

The total J is then obtained by summing dJ over all the elements on the path. Both rectangular and circular paths may be specified in the post-processor - if the former then a modified integration scheme is adopted at corner elements where the direction of the path changes within an element.

If an area integral within a path is required, for example for thermal loading in equation (5), then all four integration points are used in the evaluation of the integral (in a similar manner to the evaluation of the element stiffness matrix in ANSYS).

For the J-integral the strain energy density W in equation (3) is computed by summing over the M substeps of the ANSYS plastic analysis. W is given by the trapezoidal rule as described by Bakker [12]

$$W = \frac{1}{2} \sum_{m=1}^{M} (\sigma_{ij}^{m} + \sigma_{ij}^{m+1})(\varepsilon_{ij}^{m} - \varepsilon_{ij}^{m+1}) \ldots\ldots(10)$$

Similar methods have been used to compute the C(t) and C* integrals using equation (8), although in this case equation (10) is not required.

Analysis

All the analyses performed in this study have been of a single-edge-notch bend (SENB) fracture toughness test specimen, shown in figure 6. This was modeled with the FE mesh in figures 7 and 8. An a/w (crack depth/specimen width) ratio of 0.5 has been chosen, and all the elements are PLANE82 plane-strain elements with four integration points. There is a line of symmetry through the crack, and therefore only half the specimen has been modeled.
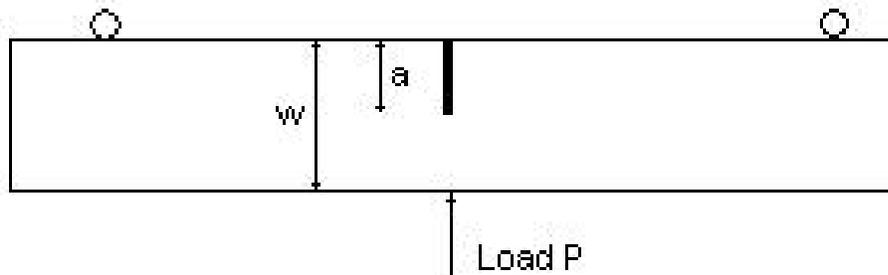

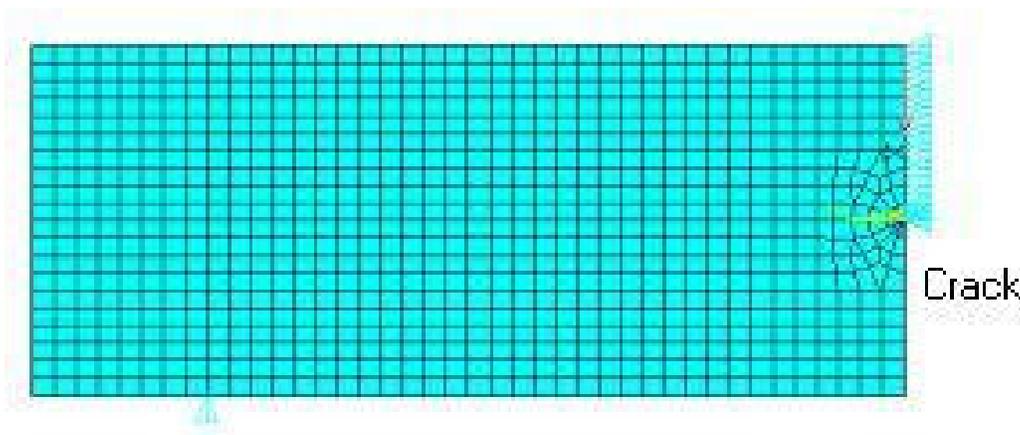
Figure 6. SENB Fracture Specimen

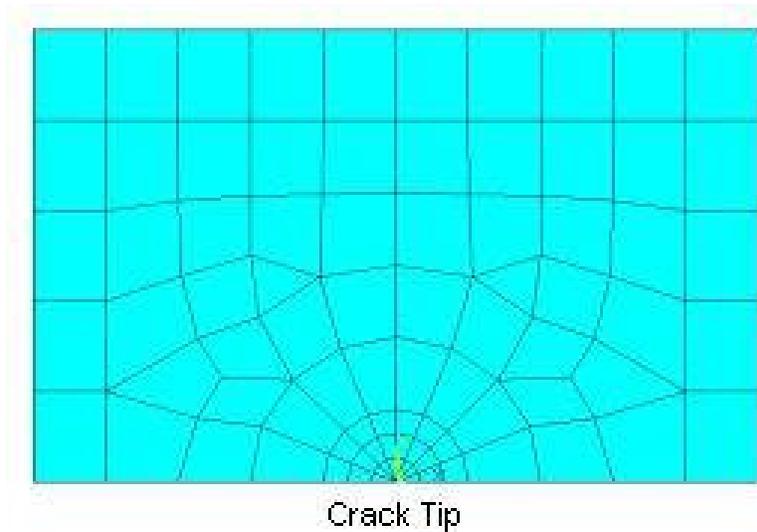**Figure 7. FE Mesh of SENB Specimen**

Crack Tip

Figure 8. Crack-tip Detail of FE Mesh

The SENB specimen has been analysed elastically using quarter-point elements, and with non-linear material using collapsed elements at the crack tip. Different material properties have been assumed: for elastic and plastic analyses a pressure vessel steel studied by Bakker [12] has been considered. The stress-strain relationship for this material is shown in figure 9. However for the high-temperature creep studies the material considered was a 1Cr-1/2Mo steel at 535°C. From Riedel [13] the power law constants in equation (7) for this material are: $n = 8.6$, $B = 5.6 \times 10^{-26}$ $MPa^{-n}s^{-1}$.
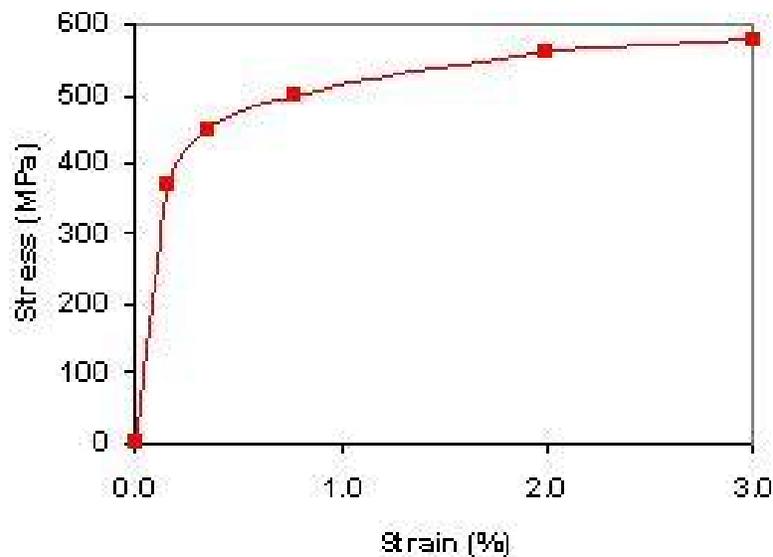

Figure 9. Stress-Strain Curve for PV Steel

The contour paths around the crack tip used to evaluate J and C* are shown in figure 10. Both rectangular and semi-circular paths are illustrated - the former in regions distant from the crack tip, and the latter close to the crack tip. Because the present work uses a half-model J and C* values computed required multiplication by two.
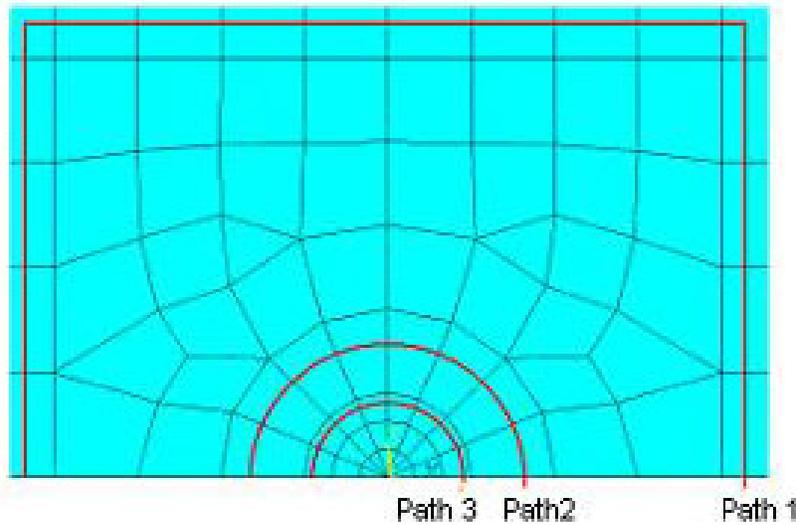
Figure 10. J and C*-integral Contour Paths

Results and Discussion

The FE model was initially validated with elastic material properties, $E = 230 \times 10^3$ MPa and $v = 0.3$, and a load P of 2.0kN. Using quarter-point elements KI was computed to be 165.3 MPa $\sqrt{mm}$ from equation (1), which is similar to the closed-form solution result from Broek [14] of 165.5 MPa $\sqrt{mm}$. The K_I value from the post-processor may be converted to an elastic J using the relationship [3] for plane-strain conditions

$$J = \frac{K^2}{E}(1-v^2)\ldots\ldots(11)$$

resulting in J = 107.5 N/m. This compares well with a computed value of J from the post-processor of 109.2 $\pm$ 0.3 N/m, the range showing the variation of J from the different paths - little path dependence is evident.

Plastic J results from the post-processor were obtained with the material properties presented in the previous section, and using the multi-linear isotropic material option in ANSYS. The computed values of J to a maximum load P of 37.0 kN are presented in figure 11. These results are compared with data from Bakker [12] and show good correspondence. Path independence of J was confirmed as over all the paths identified the mean and range of J was 23.7 $\pm$ 0.08 kN/m at a load of 25.0 kN. Once J has been calculated it may be compared with the fracture toughness J_{IC}, or the resistance curve J_R, to predict crack stability.
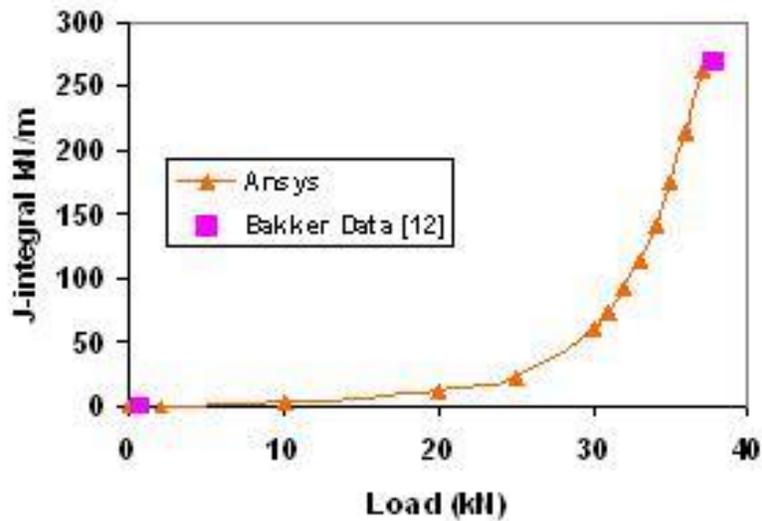
Figure 11. J-integral against Load for SENB Specimen

Finally the results from the creep analysis are summarized in figure 12. The analysis was performed with a load of 2.0 kN and was continued to a time of 1.0×10^9 s, for which the extensive creep strains are plotted in the figure Equivalent Creep Strains in SENB at t=1×10^9 s. At this steady-state time the calculated value of $C(t) = C^* = 5.70 \times 10^{-7}$ W/m2.
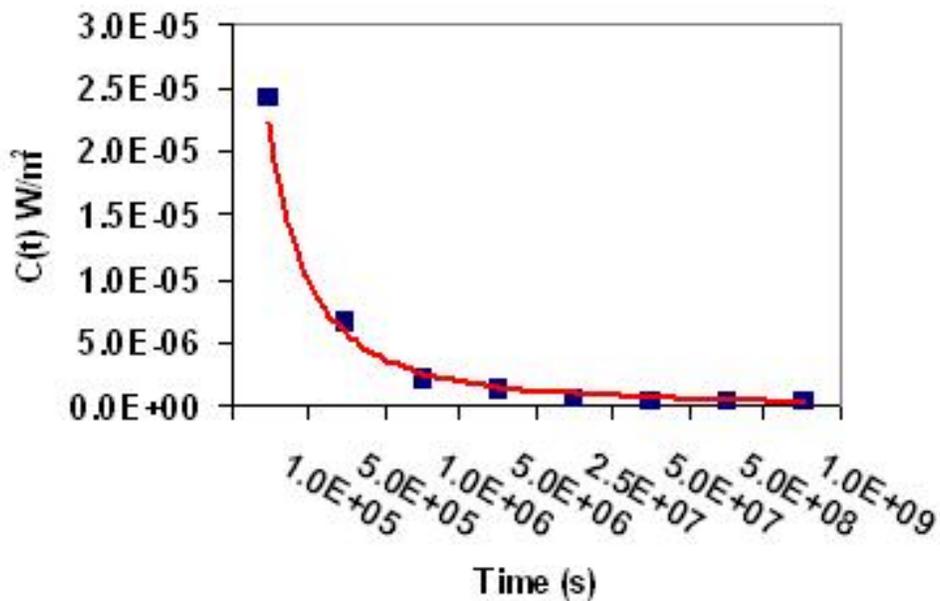

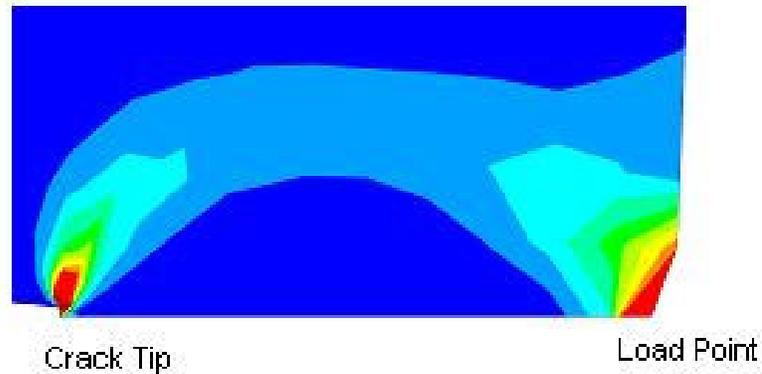**Figure 12. C(t) against Time for SENB Specimen**

Figure 13. Equivalent Creep Strains in SENB at t=1x109 s

Comparing the C(t) and C* results from the various contour paths analysed demonstrated path independence. Once C* has been calculated it may be used to predict rates of creep crack growth. For example, if one assumes growth of a crack by grain boundary diffusion, as in Witts [15], then the crack growth rate is given by

$$\dot{a} \propto (C^*)^{\frac{n}{n+1}} \ \dots\dots(12)$$

The method used in the fracture post-processor to evaluate the J and C* integrals is not the only procedure available, but was chosen as the most suitable for the present task. Also available is the virtual crack extension method introduced by Hellen [16], which is most easily implemented within the main FE program. Results for J from both these methods were compared by Bakker [12]. However in the tests on the post-processor presented here accurate results are obtained by evaluating J and C* around the defined paths.

Further work is intended, including further testing of the two-dimensional portion of the post-processor, and the completion of the three-dimensional development and verification work.

Conclusions

· Analysis of the SENB specimen has shown that the post-processor produces accurate two-dimensional results for the J and C* integrals.

· Present results for J and C* are encouraging although more verification of the two-dimensional post-processor is necessary (for example an analysis of an SENT fracture specimen for which closed-form solutions exist).

· It is intended to continue with three-dimensional development work based on the same methods.

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