Numerical Solution of Opened Steel Cylindrical Reservoir, Rigidly Connected to a Non-deformative Fundament

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Abstract

In this paper the authors present numerical solution of opened steel cylindrical reservoir, rigidly connected to a non-deformative fundament by using finite element analysis. The results from numerical solution by ANSYS 5.6 program are compared with analytical solution. Both methods are applied to the same practical example.

Introduction

Some of the most widely met structures in our construction practice are the opened Vertical Cylindrical Reservoirs (VCR), produced by using steel sheets. These are constructed mainly for storing petroleum products, and are used also as general-purpose storage tanks.

The main components in the construction of these types of reservoirs are:

- Fundament;
- Bottom;
- Cylindrical shell (wall);

which on its bottom end is firmly fixed upon solid and non-deformation fundament (Fig.1a).

The main loading on these cylindrical shells is the so-called hydrostatic pressure of the liquids that has axial symmetry. This specific requirement has justified the inference of a closed mathematical calculation intended for solving the structural analysis for the problem. In these type of structures the internal force assessment of the hydrostatic pressure varies only in terms of the height of the cylindrical shell, whereas in terms of the direction of a particular circle it remains constant.

Besides, in this type of loading significant for the determination of dimensions of the shell, the following internal forces are calculated: The vertical normal force $N_\theta(x)$ along the height of the shell, incurred as a result of its self weight; the parallel tension normal force along the height of the reservoir $N_\theta(x)$; the bending moment $M_x(x)$, and the shear force $Q_x(x)$ [1].

Analytic Solution of the Problem

The study of the stresses and strains of the thin cylindrical shell with circular cross-section is performed by the solution of the differential equation [1] for the case of VCR with constant thickness of the wall (1):

$$\omega^{IV} + 4\alpha^4.\omega = \frac{P}{D} \qquad (1)$$

Where

$$\alpha^4 = \frac{3(1-\mu^2)}{R^2\ t^2}; \qquad D = \frac{E\ t^3}{12(1-\mu^2)}$$

E- elastic modulus;

μ- Poisson's ratio;

t- thickness of the wall;

R- radius of the reservoir;

D- cylindrical stiffness of the wall;

P- hydrostatic pressure;

w- radial displacement;

This equation comprises the synthesis of all of the basic functions for the displacements, strains and stresses of a cylindrical shell for reservoirs. Its solution reduces the problem to finding out one such function of the radial displacement $\omega(x)$, which shall satisfy both the differential equation (1), and the boundary conditions on the both ends of the shell.

The establishment of this function enables as to determine the internal forces M_x, Q_x, N_θ by means of the derivatives of ω, and in particular $\omega^{II}, \omega^{III}$ and ω^{IV}, in compliance with the said theory [1].

By means of the presented classical method an example of the construction practice was solved, for the case of VCR with $D=16\ m$; $H=15\ m$; $\gamma=10\ kN/m^3$; constant thickness of the steel sheets for the cylindrical shell $t=0.012\ m$; Poisson ratio $\mu=0.3$; Elastic modulus- $E=210\ 000\ 000\ kPa$; [2], (Fig.1a).

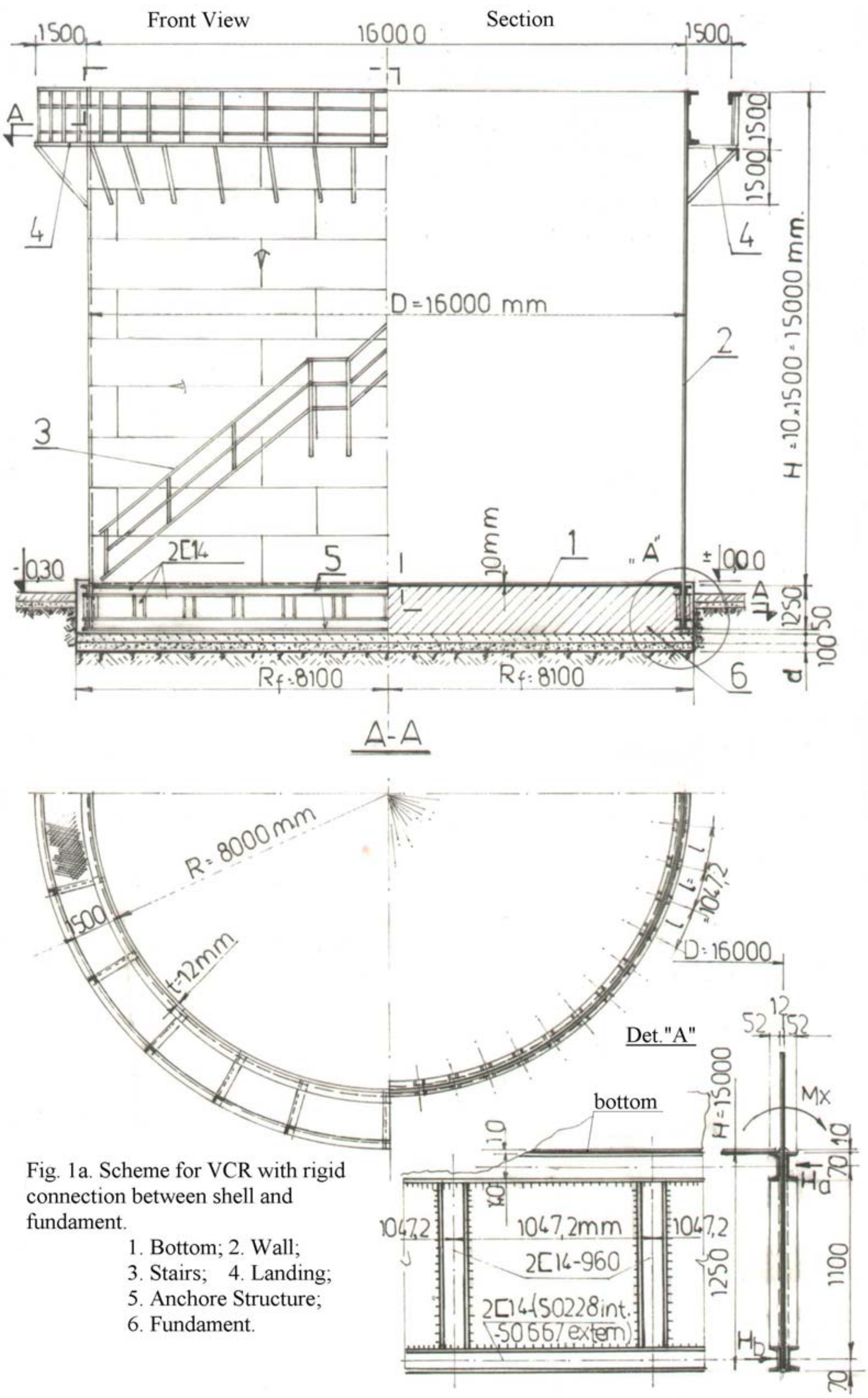

Fig. 1a. Scheme for VCR with rigid connection between shell and fundament.

1. Bottom; 2. Wall;
3. Stairs; 4. Landing;
5. Anchore Structure;
6. Fundament.

Figure 1a - Scheme for VCR with rigid connection between shell and fundament

The estimated internal forces of this tank within the cylindrical shell are shown on Fig.1b, and are as follows: bending moment along the height of the shell M_x that at the place of the rigid connection with fundament has the value of *4.277 kNm/m*, transversal internal force Q_x along the height of the shell with a value of $Q=-35.358$ *kN/m*, and normal force $N_\theta(x)$ along the height of the tank with maximum value of $N_\theta=1192.688$ *kNm/m*, at a length of *0.7 m* from the tank basis.

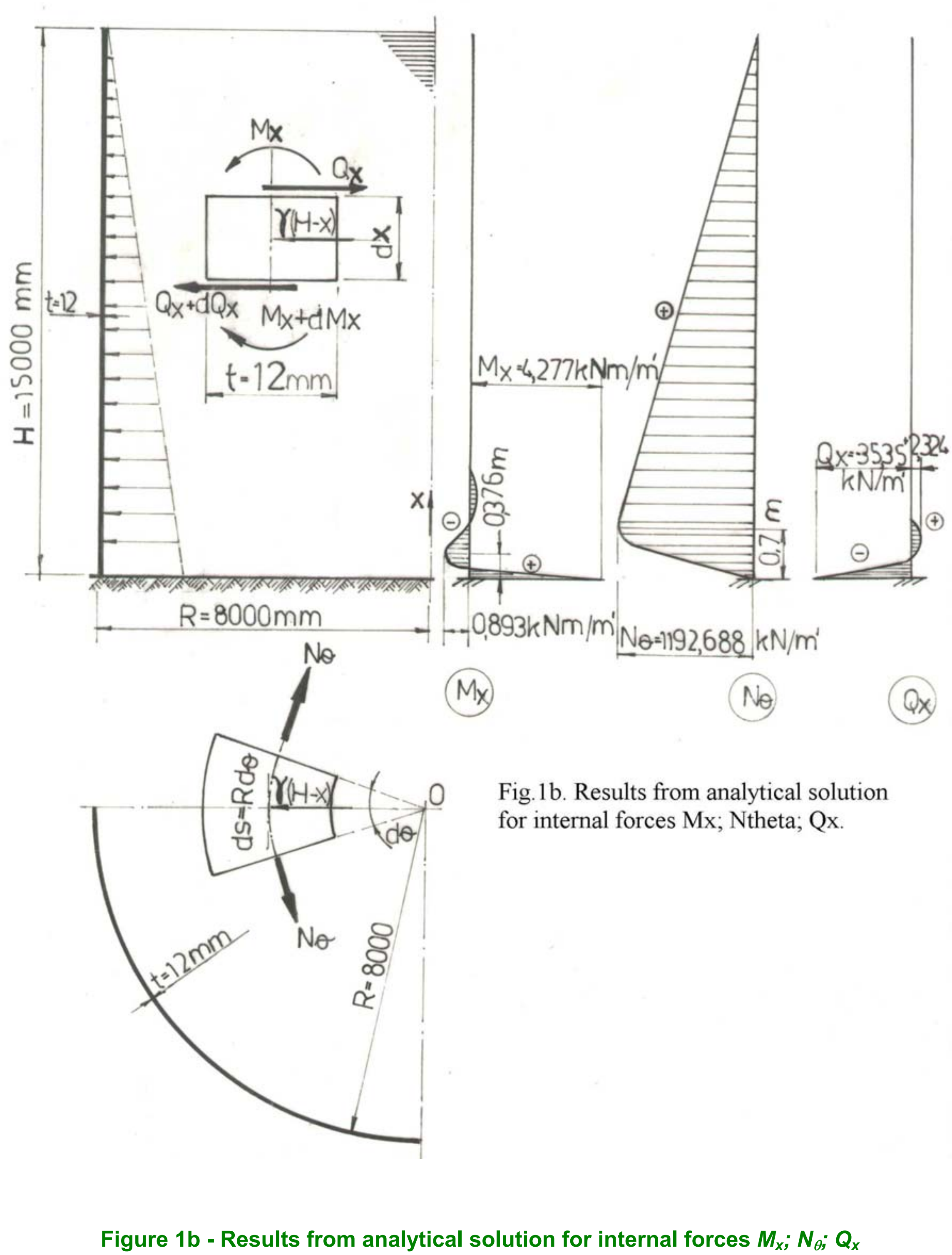

Fig.1b. Results from analytical solution for internal forces Mx; Ntheta; Qx.

Figure 1b - Results from analytical solution for internal forces M_x; N_θ; Q_x

By means of the thus achieved solution, and disregarding weight of the shell the stresses within the cylindrical shell at the place of fixed support were studied, following stresses were calculated:

$$\sigma_m = \frac{M_x}{W} = \frac{4,277}{\frac{1}{6}.1.0,012^2} = 178\,208,33\,\text{kPa}$$

$$\tau = \frac{Q}{A} = \frac{35,358}{1.0,012} = 2946.5\,\text{kPa}$$

$$\sigma_s = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{178208.33^2 + 3.2946.5^2} = 178\,281.39\,\text{kPa} = 178,28\ \text{MPa} < R = 210\,\text{MPa}$$

At the cross-section on the cylindrical wall, at a length of *0.376 m* from the basis is obtained the maximum negative bending moment having the value of M_x=*-0.893 kNm/m*. At this point the shear force is Q_x=*0*, and the normal force has the maximum value: N_θ=*1192.688 kNm/m.*

On the grounds of the said internal forces the calculated stresses are as follows:

$$\sigma_M = \frac{M_x}{W} = \frac{-0.893}{\frac{1}{6}.1.0,012^2} = -37\,208.33\,\text{kPa}$$

$$\sigma_N = \frac{N}{A} = \frac{1192.688}{1.0,012} = 99390.667\,\text{kPa}$$

$$\sigma_s = \sqrt{37208,33^2 + 99390,667^2 + 37208,33.99390,667} = 122\,315.67\,\text{kPa} = 122.315\text{MPa} < R = 210\,\text{MPa}$$

With the assigned values of the stresses, the cylindrical shell is rigidity secured against hydrostatic pressure.

The main objective of the present study is, however to illustrate the implementation of the method of the finite elements upon this well developed problem from a theoretical point of view, following which we shall compare the thus achieved results from both of the implemented methods.

Numerical solution for Reservoir

With the numerical solution of the formulated problem we shall implement the most universal method known to the Structural Mechanics, and in particular the Finite Element Method (FEM) [3]. In case of implementing this method the studied shell is meshed into finite shell elements. The implementation of this method with the solved problem has been illustrated with the ANSYS 5.6 Program.

For this objective has been used shell element – *Shell 61*, that has four degree of freedom at each of the nodes *i* and *j*; translations along the *x*, *y* and the *z* axes, and rotation around the *z* axis (Fig.2).

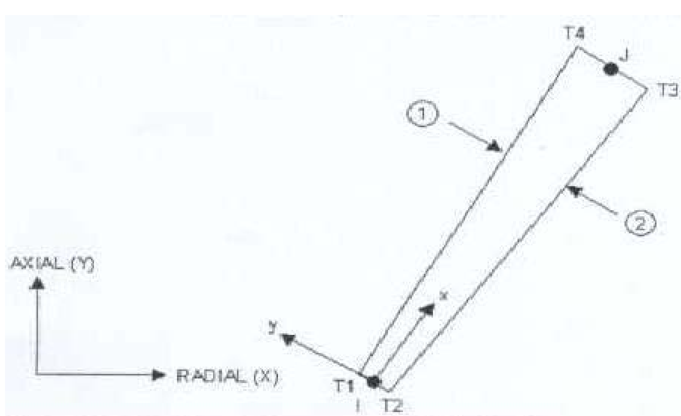

Figure 2 - Axi-symmetrical Shell element-SHELL 61

The studies performed by implementing the FEM showed that the use of a shell element with dimensions *5x5 cm* achieves quite the same results in comparison with those, achieved by analytical means. The diagrams M_x, N_θ, and Q_x, achieved by implementing the FEM are shown on Fig.3, Fig.4 and Fig.5.

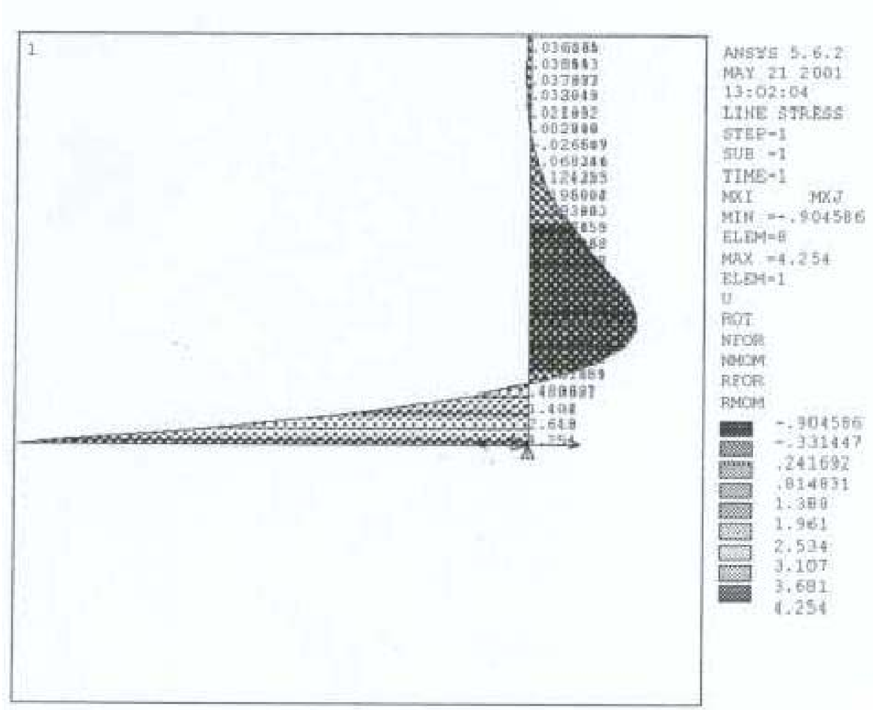

Figure 3 - Bending Moment M_x- diagram

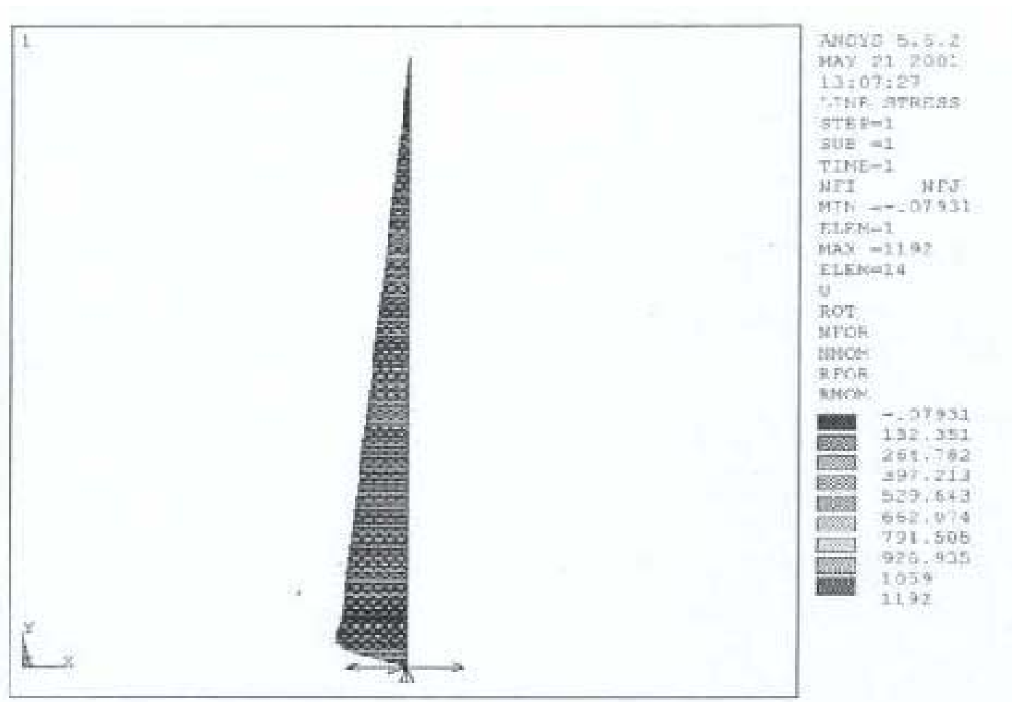

Figure 4 - Normal Force N_θ- diagram.

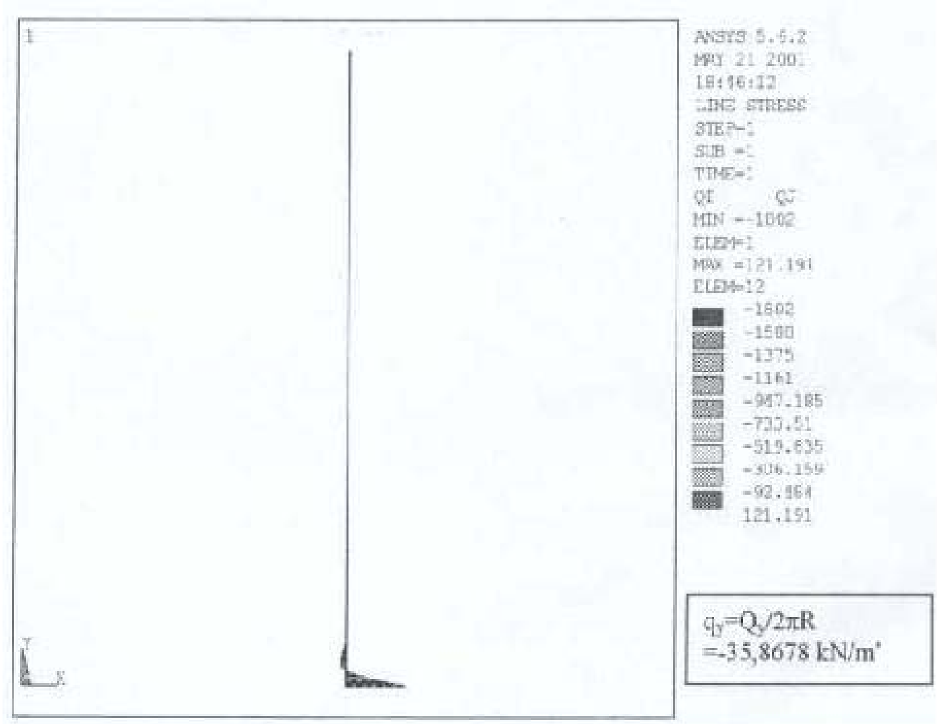

Figure 5 - Transversal Force Q_x- diagram

Conclusion

In this study is performed a successful trial of implementing the FEM by means of the ANSYS 5.6 Program for dimensioning of Opened Cylindrical Reservoir in terms of hydrostatic pressure, fixed upon non-deformation basis.

By means of a few parametric studies for the dimensions of the finite elements it was shown that by the implementation of the FEM for shell with dimensions of *5x5 cm* were achieved almost the same external forces results that were achieved by implementing the analytical method [1].

References

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