A Comparative Study of the Mechanical Response of Various Numerical Models of the Human Lumbar Intervertebral Disc

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Abstract
The present paper compares various numerical models which simulate the mechanical behaviour of the human lumbar intervertebral disc under static and time-varying uniaxial compressive loads, with the aid of the finite element method. Concerning the geometry and the mechanical properties of the disc, numerical values from the literature are adopted after suitable elaboration. All models consist of four distinct volumes, corresponding to the portions of the disc: The upper and lower endplates, the annulus and the nucleus. The differences between the models are focused to both the geometry of the various portions as well as to the mechanical behaviour of the material of each portion. The loads exerted simulate the typical daily activities of the human lumbar disc. The results of the analysis of all three models are in qualitative agreement with existing experimental data, with the most sophisticated one being closer to the reality.

Introduction
Bioengineering problems have been approached by researchers using different methods. The analytical method was never popular due to the complexity of the structures. The method most widely used in the early period of biomechanics was the experimental one, more often in vitro and rarely in vivo. The lack of specimens and the alteration of the mechanical and physical properties of the ones removed from their environment showed the inflexibility of the in vitro testing, while the obvious difficulties and the moral limitations made the in vivo experiments unpopular. It was only during the last two decades that the development of more powerful computing systems rendered numerical methods and especially the finite element analysis, one of the most widely used tools for the simulation of biomechanical problems.

The intervertebral disc is probably among the most investigated problems in musculoskeletal bioengineering, since it is one of the most complicated structures of the human body. The association of the disc with very common symptoms, such as low back pain, feeds the interest for fully understanding the mechanical behaviour and function of both the normal and the degenerated disc.

The human intervertebral disc consists of four regions (Fig.1): The upper and lower cartilaginous endplates, the nucleus pulposus and the annulus fibrosus. The endplates are the boundary between the adjacent vertebrae and the disc and many researchers consider them as part of the vertebrae. However the fact that the endplates are more strongly attached to the disc makes the authors of the present work to consider them as part of the disc, although such an approach introduces additional difficulties in the numerical simulation.

The nucleus pulposus is the gel-like inner portion of the disc and it may contain up to 88% by weight of water, while its dry weight contains 65% proteoglycans and 20% collagen. It forms a loose meshwork of randomly distributed collagen fibers in a proteoglycan matrix. The outer portion of the disc is the annulus fibrosus and it may contain up to 78% by weight of water. The dry weight of the annulus is 60% collagen and 20% proteoglycans. The boundary between these two regions is not clearly defined but is recognized as a transition from a highly hydrated area to a less hydrated one. The annulus consists of about twenty lamellae of coarse collagen fibers, arranged in a criss-cross configuration. This structure forms a strong
ligament which helps keep the nucleus intact and connects the upper and lower endplates. Finally, the endplates are composed of hyaline cartilage and are placed between the vertebrae and the two previously mentioned portions of the disc.

During daily activities the disc is loaded in a combination of compression, bending, and torsion. Flexion, extension, and lateral flexion of the spine produce mainly tensile and compressive stresses in the disc, whereas rotation produces shear stresses.

Many theories have been proposed for the constitutive behaviour of each region of the disc. The disc as a whole exhibits a highly time-dependent behaviour which is translated by each researcher as either viscoelasticity or poroelasticity. Viscoelasticity is usually attributed to the collagenous fibers of the annulus as well as to the large molecules that slip past one another in the collagenous matrix, whereas poroelasticity seems to explain the fluid flow in the nucleus pulposus and in the annulus bulk [1]. Taking into account the complex constitutive behaviour as well as the complexity of geometry it becomes evident that analytical solutions are inapplicable for the study of disc. In the light of the above discussion an attempt is made in the present work for a numerical analysis of the problem and more specifically for the comparative study of the results obtained using models of increasing sophistication.

Analysis

Assumptions

The study is carried out using the finite element method with the aid of the commercially distributed software ANSYS 9.0. A number of assumptions were made concerning the geometry and the mechanical behaviour of the material of the disc. It is stressed out from the beginning that the geometry adopted is rather simplified compared to the real one. However even today, there are no definite conclusions concerning the exact form of the boundaries of each portion of the disc and the problem is still the subject of intensive research. In any case, the fact that the models built for the present study, use a kidney-like cross section for the disc constitutes a great improvement compared to previously introduced models, that used either cyclic or elliptic cross sections. All models were built considering four distinct portions for the disc, which is a common approach for the modeling of the intervertebral disc.

As far as it concerns the mechanical behaviour, it is emphasized that the poroelasticity characterizing the disc is ignored, due to the limited material properties assigned to the elements which are provided by the software used. However, since poroelasticity influences mainly the transient response of the disc to the external loads, it can be considered that the conclusions regarding the static behaviour of the disc, approach reality. It should be noted however, that it is not yet generally accepted that the behaviour of the disc is described by poroelastic analysis. Many researchers simulate the mechanical response of the disc using a combination of linear elastic and viscoelastic models and this is the approach adopted herein. In any case, the validity of a model is finally judged on the basis of the proximity of its results to the experimental ones. It is hoped that using the expected enriched version of the software one will have the chance to introduce also poroelasticity and compare the respective results with the ones obtained here.

The last point that should be clarified is the one regarding the magnitude of external loads, their application on the disc and also the constraints at the boundaries. It is assumed that the disc is loaded under static uniaxial compression and the magnitude of the load corresponds to the weight of a normal male (about 850N) in the upright position. In addition, a time-varying loading program (transient analysis) was exerted on the disc as it is described in next sections, in an effort to simulate the daily activities of the human disc. The point of application of the load is assumed to be the projection of the centroid of the disc on the upper surface of the upper endplate (Fig.4(B)), in order to avoid parasitic bending moments since a single disc is studied here instead of a spinal unit or the spine as a whole. On the other hand, in order to avoid stress concentrations, due to point loads, it is considered that the load is applied in such a way, that the upper surface of the upper endplate behaves as a rigid plane, which is justified since this is the surface in contact with the adjacent upper vertebra (which is a rigid body compared to the disc). For the same reason the lower surface of the lower endplate is considered as rigidly fixed, with all degrees of freedom (translational and rotational) restricted.
**Description of the Models**

All three models introduced consist of four distinct volumes, corresponding to the four regions of the disc namely the cartilaginous endplates (upper and lower), the annulus fibrosus and the nucleus fibrosus (Fig. 1). The basic geometric difference between the first two models is that, while for the first one the nucleus is of constant kidney-like cross section along the height, for the second one the cross section changes following a parabolic law (Figs. 2, 3). However for both models the annulus has constant curvature along the radial direction. This restriction is removed in the third model for which the curvature of the innermost surface is compatible to the respective one of the kidney-like, spheroid shape of the nucleus, while for the most external surface it becomes zero (Fig. 4). In addition, the annulus in the case of the third model is assumed to consist of ten successive co-centric volumes of the same thickness which however varies along the height of each volume, as it is shown in Fig. 5(a). This partition was adopted in an effort to simulate in a more realistic manner the fibrous character of the annulus considering the matrix and the fibers as independent entities, rather than as a simple orthotropic material, which is the case for the first two models. The geometrical details of the three models are summarised in Table 1.

![Figure 1](image1.png)

**Figure 1.** Sagittal (a) and transverse (b) illustrations of the structure of the intervertebral disc., NP = nucleus pulposus, AF = annulus fibrosus, VB = vertebral body, EP = end-plate, PLL = posterior longitudinal ligament. [2], (c) Plan view of normal human lumbar intervertebral disc [3].

![Figure 2](image2.png)

**Figure 2.** The discretization of the regions of the first model of the disc: (a) The nucleus pulposus, (b) The annulus fibrosus and (c) The cartilaginous vertebral endplates.
Concerning the constitutive behaviour, the endplates are assumed to behave as linear elastic bodies, while the nucleus is considered as a viscoelastic material described by a nonlinear function obtained with a curve-fitting procedure, for all three models. Regarding the annulus, for the first two models, it is assumed that it behaves as a laminated composite elastic material consisting of successive transversely isotropic layers in a $\pm 30^\circ$ criss-cross configuration (with respect to the horizontal level). For the third model, a more sophisticated structure is considered for the annulus: The outer upright surfaces of the ten co-centric volumes mentioned above are assumed to be laminated shell elements (Figs.5(b,c)), while the volumes themselves are of viscoelastic character, which is described by a nonlinear function obtained again with a curve-fitting procedure. It is clarified here that the fact that the annulus consists of fibers and matrix in a continuous manner could not be taken into account for all three models. An equivalent transversely isotropic material was considered, the mechanical properties of which were obtained using the theory of fiber composite materials [4, 5]. The calibration of the model concerning the relative amount of fibers with respect to the matrix was carried out using existing experimental data by Tyrell et al. [6], concerning the vertical displacement during compressive loading. These experimental results were obtained in vivo by measuring the relative displacement of characteristic points of the spine for a number of volunteers who
were remaining recumbent for a time interval and then in the upright position for the remaining time. (Obviously, displacement values regard the spine as a unity and a reduction should be done for each disc according to its height and perhaps its relative position in the spine). Following this procedure, the percentage of fibers was determined equal to 82% for the first two models and 92% for the third one. Especially for this model the amount of fibers results to a very thin structure which in turns (and taking into account both the fibers and the remaining matrix) assigns to the annulus as a whole a very low modulus of elasticity. To overcome this problem it was necessary to increase accordingly the modulus of elasticity of the matrix, compared to that obtained from literature, in order to avoid increasing the width of the element, something that would violate the assumption of plane stress (Table 2). Such a procedure is commonly adopted in numerical simulations, since the alternative solution (accurate determination of the percentage of fibers from experimental data) yields results subjected to criticism, due to the uncertainty of the original data concerning the width of the fibers and the distance between them. This is clearly concluded from literature where one meets values for the fiber’s cross-section equal to 0.09-0.23mm² [7] or even up to 3mm² [8]. The mechanical properties finally adopted for all three models are recapitulated in Table 2.

Figure 5. (a) The 10 co-centric volumes used for the modeling of the annulus for the third model, (b) The discretization of the surfaces using the layered shell elements shown in (c).

As far as it concerns the time variation of the mechanical properties, it was obtained using experimental results from literature [6], for all three models. Especially for the third model, the main question that should be answered was the contribution of each portion of the disc to the viscous character of the structure. Various scenarios were studied and it was concluded that optimum results are obtained from the model for which the nucleus contributes with its maximum possible viscous component and only the remaining overall viscous character is attributed to the annulus’ matrix. Further it was assumed that the variation of shear and bulk modulus versus time were of the same nature. Various functions of exponentially decaying form were tested until convergence to the experimental results was achieved.
Table 1. The dimensions used for the models (M1=model 1, M2=model 2, M3=model 3).

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<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
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<td>0</td>
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<td>0</td>
<td>0</td>
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<td>17</td>
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<tr>
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<td>0</td>
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<tr>
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<tr>
<td><strong>Height (mm)</strong></td>
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<td>17</td>
<td>25</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td><strong>Nucleus at half height</strong></td>
<td>28</td>
<td>32</td>
<td>32</td>
<td>36</td>
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</table>

Table 2. The mechanical properties of the components of the disc found in literature [9, 10] and the mechanical properties used for each model after suitable elaboration [4, 5].

<table>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Nucleus</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>0.45</td>
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<td>4.2</td>
<td>-</td>
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<td>0.45</td>
<td>Yes</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Annulus as a composite material</td>
<td>142</td>
<td>37</td>
<td>34 (properties of the layers corresponding to 82% of fibers per weight)</td>
<td>162</td>
<td>66</td>
<td>46 (properties of the layers corresponding to 92% of fibers per weight)</td>
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<td>-</td>
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<td>v_ii=0.25</td>
<td>v_ii=0.24</td>
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*: Literature

The endplates and the nucleus for all three models are discretized using the Solid186, a three-dimensional 20-node structural solid element which is suitable for irregular meshes and has special features for viscoelasticity adopted for the nucleus, concerning the annulus, for the first two models use was made of the element Solid191, a three-dimensional 20-node layered solid element. Each element consists of ten layers, inclined by ± 30° successively and two elements are used to fill the volume of the annulus radially, resulting to a twenty-layered structure. For the third model, the annulus consists of ten co-centric volumes. The ten upright areas of them are meshed using the Shell99, an 8-node linear layered structural shell, with two layers each, in an effort to model the twenty layers of fibers of the biological disc. The ten volumes of the annulus are meshed with the Solid186, which makes it possible to assign viscoelastic properties to the annulus as well. In the case of the third model, a finer mesh was used for the static loading, while a more coarse mesh was considered for the time-loading program for time-saving. The final number of elements
used for the meshing is a compromise between the accuracy of the results and the CPU running time of the model as it was concluded from a series of preliminary analyses.

All solid elements mentioned so far have three degrees of freedom per node, allowing the models to be subjected to forces only. In order to overcome this restriction, the surface where the loads are applied, is meshed in addition with the Shell93, an 8-node structural shell with six degrees of freedom per node, allowing the application of moments on nodes of that surface. This mesh is superimposed to the one of the previous paragraph, rendering the model more flexible. The numerical data for the three models are recapitulated in Table 3.

Concerning the boundary conditions, all degrees of freedom are constrained at the lower surface of the lower endplate, while for the upper surface of the upper endplate, a different technique is used to simulate the transmission of loads from the adjacent upper vertebra: All nodes of this surface are enslaved for the translational degrees of freedom, with respect to the node where the loading is applied. The same constraints and loading conditions are applied to all three models in an effort to make their results and response comparable to each other.

Table 3. The numerical data for the three models.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3- for static</th>
<th>Model 3- for dynamic</th>
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<td>154386</td>
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<tr>
<td>Total time for 1 iteration (hours)</td>
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<td>0.001</td>
<td>6</td>
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</table>

Analysis, Results & Discussion

The results for the static loading case are plotted in Figs.6-13. In Fig.6 the deformed shape of each model is shown together with the respective vertical displacements. As it was expected, the maximum displacement is the same for all three models (equal to about -0.56mm), since this parameter was the one used for the calibration of the models. The horizontal displacement is greater in the case of the second model something that sounds reasonable since the Poisson’s ratio adopted is almost twice the respective one of the third model.

In Fig.7 the equivalent von Mises stress is plotted at the mid-height plane. The most interesting conclusion is that while for the first two models the maximum stress developed is almost the same equal to about 2.5MPa (appearing at the interface between the annulus and the nucleus), for the third one the maximum stress developed exceeds 4MPa and it appears at the fibers of one of the intermediate volumes of the annulus. Such a conclusion indicates increased stiffness of the third model along the horizontal directions something that is compatible to the respective conclusions concerning the increased horizontal displacements observed in the first two models. Concerning the behaviour of the various parts of the disc, the nucleus appears to be almost stress free for all three models as it is expected, due to its gel-like nature.

Figure 6. The deformed shape and the vertical displacements of each model: (a) model 1, (b) model 2, (c) model 3.
The same quantity (equivalent stress) is shown in Fig.8 in vertical plates along the maximum horizontal diameter. It is to be noted that the points where the maximum stress appears are not the same for all models. For the first two of them the stress is maximized in the vicinity of the endplate, while for the third one extreme values appear also in the fibers of the annulus. Again the relative maximum value corresponds to the third model. The distribution is symmetric with respect to the sagittal plane, the maximum value is equal to about 1.3MPa and strong fluctuations are observed in the descending branch of the respective Fig.8(c), obviously due to the existence of successive co-centric layers in the annulus of the third model. Similar conclusions can be drawn from Fig.9, where the equivalent stress is plotted in the sagittal plane.
The variation of the stress components along the maximum diameter are plotted in Figs.10(a,b,c) for all three models. It can be seen that only the $\sigma_{zz}$ component attains relatively high values while the shear stresses are almost negligible. The $\sigma_{zz}$ stress component is constant in the nucleus as it is expected and then its value decreases rapidly to a minimum one depending on the specific model. The minimum is reached in the volume of the annulus for all models and then it increases until one reaches the outer boundary of the disc. The extreme value is maximum for the third model while the smaller one corresponds to the second model. Of importance is also the variation of the $\sigma_{yy}$ component which is again constant (of compressive nature) within the nucleus and then it changes its sign becoming tensile at the outer boundary. Since the magnitude of $\sigma_{xx}$ within the nucleus is exactly equal to that of $\sigma_{yy}$ one concludes that the stress state of the nucleus approaches that of almost hydrostatic compression. In any case however due to the nature of the nucleus the respective stress field is very weak compared to that of the annulus.

![Figure 10](image1.png)

**Figure 10.** The stress tensor components at a path at the mid-height plane along the maximum diameter for each model: (a) model 1, (b) model 2, (c) model 3.

![Figure 11](image2.png)

**Figure 11.** The equivalent von Mises stress distribution at a path at the mid-height plane along the maximum diameter for each model: (a) model 1, (b) model 2, (c) model 3. Figures (d) and (e) show details of figure (c).

Concerning the distribution of the vertical strain along the maximum diameter, it is observed from Fig.13 that it is almost constant in the nucleus and then it tends to a constant value equal to about -0.02 for the first and the third model while for the second one this value tends to zero as it approaches the outer boundary. The variation of $\varepsilon_{zz}$ just described indicates that the initially horizontal planes of the disc do not remain plane but rather they are deformed in the loaded state since the response of the various portions of the disc to the same external load changes according to the specific properties assigned to each one of them.
Finally, the results for the time-varying loading case (Fig.14) are shown in Fig.15. In this figure the circadian variation (24-hour cycle) of the loss of height of the disc as obtained from the present numerical analysis is plotted for all three models, together with the respective experimental data by Tyrell et al. [6], reduced for the L3-L4 disc. It is clear from this figure that only the third model is capable to predict the response of the disc to the external load. The first two models yield an almost insensitive behaviour of the disc versus time, as it is to be expected since for both of them viscoelastic nature was attributed only to their nucleus. Concerning the third model it is clearly seen that it approximates in a very satisfactory manner the overall time dependent behaviour of the disc, justifying the assumptions on which its construction was based. It is indicated however that small deviations are observed at the end of the
unloading period. More sophisticated models should be employed based perhaps on different constitutive laws for the loading and unloading portions of the load program.

![Loading Program](image)

**Figure 14.** The time-varying loading program used for the transient analysis of the models.

![Response Comparison](image)

**Figure 15.** The response of all three models in comparison to the experimental results by Tyrell et al [6].

**Conclusions**

Three numerical models of the human lumbar intervertebral disc of increasing sophistication were studied in this paper. The main advantage of these models lies in the accurate simulation of both the structure and the shape of the various portions of the biological disc (at least with respect to the clinical data available nowadays), as well as of the constitutive behaviour of the material of these portions. The need to study different models emanates from the fact that both the exact geometry and constitutive behaviour of the disc...
are not clearly defined yet and the opinions of experts still stand apart. On the other hand the complexity of a model is restricted by the fact that it must also be flexible and not extremely time consuming.

The models were subjected to both static and time-varying loadings corresponding to typical activities of a human and the stress state developed was described in terms of displacements, stress components along characteristic paths and the equivalent von Mises stress. The results concerning the time-varying load case were compared with some of the limited available experimental data and the agreement of the third model was proved satisfactory.

Concerning the relative behaviour of the three models it was concluded that for the first two of them the differences are relatively small. It is the third model for which the stress field developed diversifies being more severe compared to the respective field of the first two models. Such a behaviour could be explained if one takes into account the restrictions imposed to the radial displacements by the fibrous structure of the annulus, as it is adopted in the third model.

Coming to an end it should be stressed out that in order to simulate the real behaviour of the disc using numerical analysis, models of increasing sophistication must be used and if possible models adapted to specific groups of people since the morphology and properties of the disc vary with age, sex, type of activities, degenerations, etc. Although such an analysis is far from the current computational tools available, improvements could be proposed, the most important of which include more accurate:

- simulation of the geometry of the disc and its parts, using CT’s pictures;
- reproduction of the inner structure of the parts of the disc and especially that of the annulus, consisting of fibers and matrix as independent structural elements;
- constitutive models including poroelasticity for the description of the disc, especially at the interface between the nucleus and the annulus;
- simulation of the gradual variation of the mechanical properties from the nucleus to the annulus, instead of considering a well defined interface.

Preliminary studies with models including some of the above improvements indicate that the time required for their analysis increases dramatically rendering them hard to use and non-flexible especially for parametric studies which require increased number of iterations. In any case the final decision should be a compromise between accuracy and flexibility and the results are judged according to their proximity to experimental reality.

References

3. http://www.emba.uvm.edu/~iatridis/research